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The triaxial load response of grain

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THE TRIAXIAL LOAD RESPONSE OF GRAIN

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The triaxial load response of grain

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INTRODUCTION

This dissertation is composed of three published papers, material which is in preparation for publication, and supplementary appendices.

Parts I and II, 'Dilatancy as the cause of overpressures: an hypothesis' and 'Dilatancy as the cause of overpressures: experimental evidence', were published in the reprint and discussion volume, respectively, of the International Conference on the Design of Silos for Strength and Flow, which was held in Lancaster, England in September 1980.

Part III, 'Frictional and stress-strain characteristics of selected agricultural grains as indicated by triaxial testing' is being published in the Proceedings of the Powder and Bulk Solids Conference '81, which was held in Chicago, Illinois in May 1981.

Part IV, 'Frictional properties and stress-strain relationships for use in the finite element analysis of grain silos' is currently in preparation for publication in the Journal of Powder and Bulk Solids Technology.

The appendices include a detailed literature review on silo design stresses, details of the triaxial test apparatus, scanning electron micrographs of grain surfaces and details of the numerical analyses.

A Note on Terminology

In this thesis, the term 'dilation' is used in its soil mechanics sense, i.e. a volume increase. In engineering mechanics terminology, dilation may refer to a volume increase or decrease.

The grain known as corn in North America was referred to as 'maize', to avoid confusion at the international conferences, since in many English speaking countries 'corn' refers to the local cereals which may not include maize.

OBJECTIVES

The overall objective of this research, of which this dissertation forms a part, is to investigate the mechanistic behavior of granular materials which are commonly stored and handled in bulk. These investigations will evaluate parameters necessary for the improved design of storage and handling facilities.

The objectives of the work reported here are:

i) to review the literature on design stresses in silos,

ii) to design and construct a low stress triaxial test apparatus suitable for evaluating the mechanistic behavior of agricultural grains,

iii) to evaluate the stress-strain relationships for four agricultural grains (maize, wheat, barley and oats), for use in existing finite element models of grain silos, and

iv) to test the hypothesis that agricultural grains, like cohesionless soils, may undergo dilation (volume increase) upon shearing.

PART I. DILATANCY AS THE CAUSE OF OVERPRESSURES: AN HYPOTHESIS

Abstract

It is well-known that the lateral loads exerted by granular material within a silo during unloading are considerably higher than the loads produced when the silo is filling. No coherent mechanistic explanation has been advanced to explain this behavior. This paper offers the hypothesis that dilation is the cause of overpressures. Dilatancy is the tendency for a dense granular material to expand in volume when sheared. If a dilatant material is held at constant volume whilst being sheared, an increase in lateral stress results. It is suggested, by analogy to sands, that dilation can occur in any granular material, provided the stress levels are appropriate, and that as the material shears in the unloading process it tends to increase in volume, thereby causing overpressures. It is further hypothesized that the stress-strain characteristics of granular material will depend upon the stress path to which the material is subjected, and the best method for evaluating the behavior of the material is triaxial testing. Experimental verification of the hypotheses may lead to a more rational silo design.

The Problem

Within the last few decades, a number of silos containing granular material have failed, with structural distress ranging from spalling and cracking to total collapse of the structure. In many cases, these failures occurred during the unloading of the silo, not when the silo was being filled nor under the static load of the stored material. A recent example of such a failure occurred on September 18, 1979 in Bode, Iowa, U.S.A., when a 45 m high, 2500 tonne capacity, concrete silo filled with maize collapsed while the silo was being emptied. The cost of the damage from this failure was estimated in excess of $500,000. Since the 1890s, it has been known that the lateral loads against the walls of silos during emptying may be greater than the loads developed when the silo is filling. Most of the research on this topic has been by measurements on model or full-scale silos or by theoretical analyses based upon assumptions regarding the mechanistic behavior of the granular material. Little attention has been given to characterizing load response of grain and other particulate material, thus there is no coherent explanation or theory of "overpressures" developed when a granular material is emptied from a silo.

Design Stresses in Silos - an Overview

The following is not intended as a complete review of the problem of estimating the overpressures in silos, but is intended to show the important ideas which have shaped today's design practice, and to synthesize some of these concepts. A more complete review may be found in appendix A.

Early designers, believing that a granular material behaved like a fluid, designed for hydrostatic pressures calculated on the basis of the material's apparent fluid density. This conservative procedure was later modified by applying a coefficient of lateral stress to the vertical stresses, as is the practice in soil mechanics. This modification adequately described the horizontal stresses developed in shallow bins and is still used today for such structures.

Janssen (1895) provided the first analytical approach to calculating the static stresses within a granular mass contained in a silo or deep bin, by recognizing that vertical frictional forces are mobilized between the walls and the material. Janssen summed all the forces acting on a horizontal element within the granular mass and set them equal to zero. The integration of the first order differential equation which results from the statement of static equilibrium gives an expression for vertical pressure at any depth. Koenen (1896) suggested that the lateral stress be determined by multiplying the vertical stress by the active case lateral stress ratio, K_a, i.e. $K_a = (1 - \sin\phi) / (1 + \sin\phi)$, where ϕ is the internal friction angle of the granular material. The derivation of Janssen is identical to the equation derived by Marston (Marston and Anderson, 1913) for calculating the soil loads on underground conduits.

Two years later Airy (1897) used a sliding wedge theory, similar to that used in soil mechanics for retaining wall design, to compute lateral stresses in silos. A wedge of the granular material is assumed to have started sliding along a plane. The strain mobilizes intergranular friction along the shear plane and between the silo wall and the material. From the equations for static equilibrium, the critical failure plane can be defined. Finally, the lateral stress per unit of perimeter can be calculated for any depth. Airy's theory does not require the definition of a lateral stress ratio.

Shortly after the publication of Janssen's equations, Prante (1896) reported that larger lateral stresses developed upon unloading of bins. Some early experimenters reported similar increases, whereas others reported no stress difference upon emptying. In his still highly respected textbook Ketchum (1919) concluded, from the results of his own experiments, that there was no appreciable increase in lateral stress due to unloading from a concentric discharge gate.

Jannsen's equation was recommended and used widely by authors and designers, and seems to have been satisfactory until the 1930s when refinements in design procedures and construction methods led to a reduction in safety factors. Later Jàky (1948) developed a double-functioned solution for design purposes in which the "at rest" lateral stress ratio, $K_o = 1 - \sin\phi$, was used. Structural failures renewed the interest in stress prediction and much experimental work was undertaken in continental Europe. Takhtamishev, as cited by Turitzin (1963), noted that two major flow types existed when a bin was unloaded. The first type, called funnel-flow, was characterized by material proceeding to the discharge gate via a central funnel which is supplied only by material from the top of the bin. The second type, call mass-flow, was characterized by the entire mass moving simultaneously toward the discharge opening. Takhtamishev found that only mass-flow, which occurred in loosely packed bins, was responsible for the greater lateral stresses. This partially explained the inconsistencies reported by Prante and Ketchum.

Extensive model tests were the basis for a theory for deep bins in which the lateral stress distribution was hyperbolic (Reimbert, 1943). M. Reimbert and A. Reimbert (1956, 1976) recommended either increasing the lateral stress values by a dynamic coefficient to allow for increased stresses during discharge in mass-flow bins, or the installation of their patented device for ensuring funnel-flow in deep bins. Isaacson and Boyd (1965) and Isaacson (1971) have shown that Janssen's theoretical solution and Reimbert's semi-empirical solutions are both special cases of a more general solution for which they have offered a computer-based mathematical model.

Kötter in 1899, as cited in Jenike and Johanson (1968), suggested that active stresses developed during filling, whereas passive stresses developed during emptying of silos. Thus, for unloading the lateral stress coefficient is: $K_p = (1 + \sin\phi)/(1 - \sin\phi)$, i.e. the reciprocal of the active coefficient. This concept was incorporated in the stress prediction equations developed by Caquot and Kèrisel (1949) and by Ohde, 1950, and Nanninga, 1956, both cited in Jenike and Johanson (1968). None of these equations has been widely accepted, although it is interesting to note that recently Gaylord and Gaylord (1977) and Vivancos (1978) have re-introduced the idea of using the active lateral stress coefficient during loading and the passive lateral stress coefficient during unloading.

Jenike (1961, 1964) and Jenike and Johanson (1968, 1969) developed differential equations for static and flow loadings assuming a radial stress field, i.e. a field in which stresses increase linearly from the vertex of the discharge hopper. The greater lateral stresses developed during discharge, commonly called dynamic overpressures, were attributed to the switch from the active to passive stress field. These theories were modified by assuming that the recoverable strain energy of the stored material below the switch would be minimized (Jenike et al., 1973a, b,c). It was shown that the resulting equation gave the upper bound, and that Janssen's equation gave the lower bound. Jenike and Johanson recommended using a constant lateral stress ratio of 0.4 for all materials and produced dimensionless charts using this value to facilitiate the solution of their equations. Cornish (1973), however, used the discontinuity theory of Schofield and Wroth (1968) to show that higher stresses than those predicted by Jenike and Johanson are theoretically possible during mass-flow.

Modern design codes tend to recommend the methods of Janssen and the Reimberts for calculating lateral loads. Both methods incorporate the active lateral stress ratio for loading or static stresses. The American Concrete Institute's (1977) design manual recommends that lateral stresses during unloading be calculated by multiplying the active case lateral stress by an overpressure factor ranging from 1.1 to 2.0 depending upon the height and diameter of the silo. Consideration is also given to impact factors. German and Soviet codes written in the 1960s recommend the use of Janssen's equation in conjunction with dynamic overpressure factors for various portions of the bin. The latter two codes have subsequently been adopted in numerous countries.

Rationale for the Hypothesis

From the previous section, it can be seen that the mechanical behavior of the stored granular material is given little consideration in calculating lateral stresses resulting from "overpressures" during the emptying of a silo. Although the ACI code recommends that the shear strength, i.e. ϕ, the angle of internal friction, and unit weight of the granular material be measured, most designers tend to take these data from published tables. Although the Reimbert and Janssen approaches depend upon the angle of internal friction of the grain to calculate the lateral stress, the Jenike and Johanson approach reduces the significance of the strength characteristics of the stored material by using a constant lateral stress ratio.

The hypothesis offered here is that in order to predict realistically the lateral stresses that are developed when a silo is unloaded, it is necessary to understand the stress-strain characteristics of the granular material during shear. Further, it is hypothesized that dilation or a tendency for volume expansion may occur, and that the actual stress-strain characteristics of the granular material will vary depending upon the stress path which the material experiences.

The rationale for this hypothesis is that all granular materials should exhibit stress-strain and strength characteristics similar to non-cohesive soils such as sands and gravels. High density sands have high friction angles, and for most sands the Mohr failure envelope is a straight line, and the internal friction angle a constant. Very dense sands exhibit a tendency to expand in volume during shear. This volume increase, sometimes referred to as dilation, is the result of the closely packed, interlocked particles having to ride over one another as the soil is strained (Taylor, 1948; Rowe, 1962). Sands which have low density prior to loading decrease in volume during shear because there is sufficient void space into which the particles can move. For every sand there is a critical density at which the sand will experience no volume change during shear. The higher friction angles in sands with higher densities are the result of greater interlocking of the grains.

These generalizations are true for most quartz sands tested over fairly narrow stress ranges. If the stress ranges are very wide and/or the sand particles are very soft, such as a carbonate sand, the Mohr envelope is not a straight line, but tends to curve downward at higher confining stresses to produce lower friction angles at higher stresses. This is the result of deformations or crushing at individual particle contacts which decrease the effect of interlocking at the higher stresses.

One of the most useful tests in soil mechanics is the triaxial test. Triaxial testing apparatus have the capability of measuring volume change in the specimen, axial strain, and (in some special cases) lateral strain as the specimen is loaded. The standard triaxial test on a soil, called a compression loading test, consists of applying a constant isotropic stress while the specimen is loaded axially until it fails. By plotting the failure stress Mohr circles for several confining stresses, it is possible to determine the angle of internal friction, ϕ, for the soil.

The triaxial apparatus has the capability of simulating other stress conditions to which a soil (or other granular material) may be subjected. For example, if lateral strain is monitored it is possible to control both the lateral stresses and axial stress to produce anisotropic consolidation of the specimen for various amounts of lateral strain. If both stresses are increased while maintaining a condition of no lateral strain, it is possible to determine experimentally the lateral stresses in the granular material at the center of a rigid silo for the "at rest" condition. The stress-strain behavior during an active case failure can be simulated by holding the axial stress constant while the lateral stress is decreased to failure, and is referred to as compression unloading. The utilization of nonstandard triaxial tests is sometimes referred to as the stress path approach (Lambe and Whitman, 1979).

Considerable research has shown that although the Mohr failure envelope for a sand is independent of stress path, its stress-strain characteristics vary greatly depending upon the stress path. In general, the strains produced in compression loading and extension loading are much higher than the strains at equivalent stresses in compression unloading (Lambe and Whitman, 1979). It seems that other granular materials would exhibit the same sensitivity to stress path.

If a sand which has the tendency to dilate is tested triaxially at constant volume, an increase in lateral stress will occur. This type of triaxial test is accomplished by monitoring the volume of the specimen as it is loaded, and adjusting the confining stress so as to maintain constant volume. For dense sands, the confining stress has to be increased, and for loose sands the confining stress has to be decreased (Bjerrum et al., 1961). Bjerrum's constant volume tests have shown a lateral stress increase from 100 to 600 kPa during shear for dense sands.

When a silo is emptied, the granular material within the silo must shear internally in order to flow from the silo. In the case of mass-flow, which produces high overpressure during unloading, the behavior of the granular material may be very much like the constant volume shearing of sand, in which high lateral stresses are produced as a result of the material's tendency to dilate. It is recognized that in the case of grain there is an apparent inconsistency, because Takhtamishev noted that mass-flow occurs when the grain is in the loose state. Loose sands would show no tendency for dilatancy and thus no lateral stress increase upon constant volume shearing. Takhtamishev's observation does not disprove the hypothesis, because there may be a critical stress or density condition at which grain will dilate. At low stresses or densities, dilation will not occur because the large void spaces between particles and the low degree of interlocking will allow volume decrease during shear. At higher stresses the interlocking effects will be nullified by deformation at the particle contacts, with a behavior similar to that of a sand composed of soft particles. This interpretation is supported by the observation that the Mohr failure envelope of grain is often not linear, and that at high stresses the friction angle, ϕ, is lower than it is at low stresses. This may also explain in part why a grain elevator which fails when it is emptied begins to fail at an intermediate height. At great depths in the silo, the density of the grain would be high, but interlocking minimized by deformations at particle contacts. At shallow depths, interlocking would not be important because of the low density of the grain. It is recognized that arching of the particulate mass within the silo may be an additional consideration.

Summary and Conclusions

The foregoing discussion suggests that if the stress-strain and strength behavior of granular material is similar to the mechanical behavior of sands, the cause of overpressures resulting from unloading in silos may be due to dilation, and that more realistic and economical overpressure estimates for silo design may result from triaxial testing of the material. Triaxial testing should result in additional insights regarding the mechanistic behavior of granular material, because not only stress path but also particle shape, particle hardness, gradation, moisture content, method of filling, time of storage, and vibrations during storage will all influence the density of the grain, which in turn will influence its load response. These considerations should result in practical suggestions for the handling and storage of granular material.

PART II. DILATANCY AS THE CAUSE OF OVERPRESSURES: EXPERIMENTAL EVIDENCE

Abstract

The design of silos for lateral stresses has traditionally used a limiting stress analysis, in which design stresses are based upon the angle of internal friction of the stored material and the coefficient of friction of the material upon the walls of the silo. Overpressures, which result when granular material is unloaded, are accounted for by empirical overpressure factors. An hypothesis to explain overpressures on a rational basis has been advanced, which suggests that overpressures result from dilation of the stored material. This dilation occurs at incipient failure as the granular material begins to flow (Smith and Lohnes, 1980a). Recent experiments using a specially constructed triaxial test apparatus have shown that maize dilates during shear, thereby providing experimental evidence to support the earlier speculation. The tests also indicate that failure stresses are dependent upon strain rate. These results indicate that: more attention should be given to stress-strain characteristics of granular material; material handling is important with regard to stresses in silos; and silo design based upon limiting stress analysis may be somewhat unrealistic.

The Problem, Hypothesis and Rationale

Although it has been known since the 1890s that greater lateral loads or "overpressures" may develop when a silo is being unloaded, no fully coherent explanation for this phenomenon has been developed. Most research has been confined to model or full-scale studies, or to theoretical analyses based on assumptions regarding the mechanistic behavior of the stored material. Little attention has been paid to characterizing the load response of grain and other particulate material, and this is reflected in current lateral stress design procedures. It is believed that characterization of the stress-strain behavior of granular material is prerequisite to understanding the cause of the lateral stress increases, and should provide a more rational basis for silo design and material handling (Smith and Lohnes, 1980a).

It has been hypothesized (Smith and Lohnes, 1980a) that, during the unloading of a silo, dilation or a tendency for volume expansion may occur, and that the stress-strain characteristics of the granular material will depend on its stress history. The rationale for these hypotheses is that all granular materials should exhibit stress-strain and strength characteristics similar to those of noncohesive soils. It was further suggested that triaxial testing was the best method for evaluating the characteristics.

The Experimental Problem

The conventional triaxial test apparatus designed to investigate the stress-strain behavior of soils, consists of a pressure chamber in which a cylindrical test specimen is subjected to controlled stresses. The specimen, which is encased in a rubber membrane, can be subjected to a confining fluid pressure that is controlled throughout the course of the test. Simultaneously the specimen is subjected to mechanically-applied axial loads. In addition to measuring the principal stresses, the apparatus has the capability of measuring volume change in the specimen, axial strain, and (in special cases) lateral strain as the specimen is loaded (see Fig. 1). The standard triaxial test, called a compression loading test, consists of applying a constant isotropic stress via the fluid in the pressure cell, whilst the specimen is loaded axially to failure. The confining fluid pressure, i.e. the lateral stress, is the minor principal stress, and the total axial stress (the mechanically-applied stress plus the confining pressure) is the major principal stress at failure.

The triaxial test apparatus also has the capability of simulating other stress conditions. For example, if the lateral strain is monitored it is possible to control both the lateral and axial stresses to produce anisotropic consolidation of the specimen for various amounts of lateral strain. If both stresses are increased to maintain a condition of zero lateral strain, it is possible to simulate experimentally the lateral stresses in the granular material at the center of a rigid silo for the "at rest" condition. If the lateral stress is adjusted throughout the test to maintain a condition of constant specimen volume, it is possible to simulate experimentally the lateral stresses in granular material that is being sheared during the unloading of a rigid silo. The stress-strain behavior during an active case failure can be simulated with a compression unloading test, in which the axial stress is held constant whilst the lateral stress is decreased to failure. The use of nonstandard triaxial tests is sometimes referred to as the stress path approach (Lambe and Whitman, 1979).

In order to use the triaxial test to investigate the stress-strain

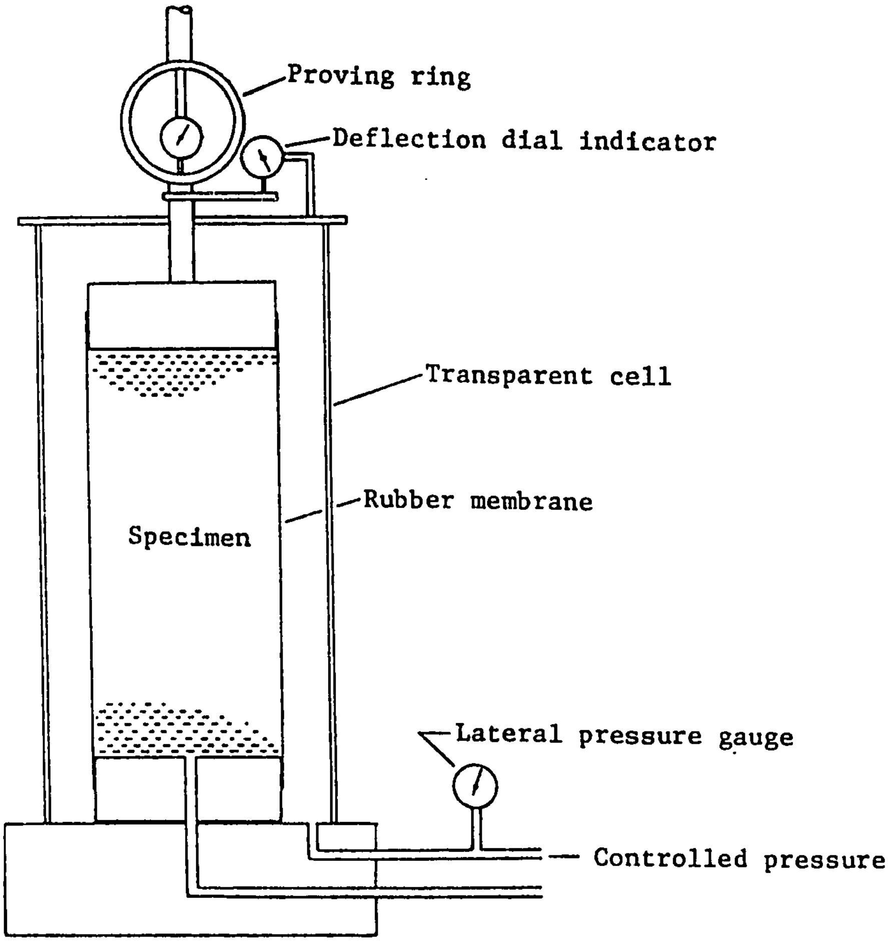

Fig. 1. Schematic diagram of conventional triaxial test apparatus

behavior of agricultural products, two major problems need consideration. First, the stress ranges encountered in the storage and transport of agricultural grains are much lower than those normally considered for soils; and second, the individual particle sizes of agricultural products are much greater than those of soils. These differences require a test cell which allows for accurate measurement of low stress levels, and which contains a large sample to minimize the effects of individual particle behavior.

Triaxial Apparatus for Agricultural Products

The special triaxial apparatus was designed to perform standard, zero lateral strain, and constant volume tests at lateral stresses up to 70 kPa. In conventional triaxial apparatus, the changes in specimen volume are monitored by observing the displacement of the confining fluid, which necessarily has to be an incompressible liquid, normally water. In the stress ranges normally considered for soil, the hydrostatic pressure difference between the top and bottom of a 305 mm tall specimen, 3 kPa, is negligible, but such a pressure difference cannot be ignored in the testing of agricultural products when lateral stresses as low as 4 kPa are encountered. Lateral strain is normally measured with a calibrated expanding hoop or caliper placed around the circumference of the sample at mid-height, and consequently only measures deformations at that point. The additional confining stress applied by the lateral strain measuring device is small, but for low stress levels should not be ignored.

The complications outlined above were circumvented by using air as the confining fluid, and by making novel alterations to the test cell for applying the confining stress, and for monitoring volume change and lateral strain in the specimen. For standard tests, the lateral stress is applied by increasing the air pressure in the cell, and venting the sample pore space to atmosphere. In the constant volume tests the lateral stress is applied in the same fashion, but is adjusted throughout the test, such that the volume of the specimen pore space, and thus the volume of the specimen, is held constant. The latter is achieved by coupling the specimen pore space to a volume monitor, which consists of a bead of mercury in a horizontal capillary tube. Any change in the specimen volume during axial loading will cause air to enter or leave the capillary tube, thereby displacing the mercury bead. Constant specimen volume is achieved by adjusting the confining air pressure, such that the mercury bead remains stationary in the capillary tube throughout the test (Fig. 2).

A unique feature of this apparatus, which makes the zero lateral strain test possible, is that the ram which enters the cell to apply the axial stress has the same cross sectional area as the specimen. The confining stress for the zero lateral strain tests is applied by partially evacuating the specimen pore space, whilst maintaining the cell air at atmospheric pressure. The volume change of the specimen is monitored indirectly by coupling the volume monitor to the cell air space. Since the cross sectional area of the entering ram and shortening specimen are equal, a constant cell air volume would create a condition of zero lateral strain for the specimen. Thus, it is possible to achieve zero lateral strain by adjusting the specimen pore space vacuum, such that the mercury bead remains stationary in the cell air volume monitor throughout the test (Fig. 3).

The proving ring, used to measure the axial load, is placed within the cell to avoid the problem of ram friction, which is significant in a conventional triaxial test machine, but greatly magnified when the ram is the same diameter as the sample. The proving ring is rigid in

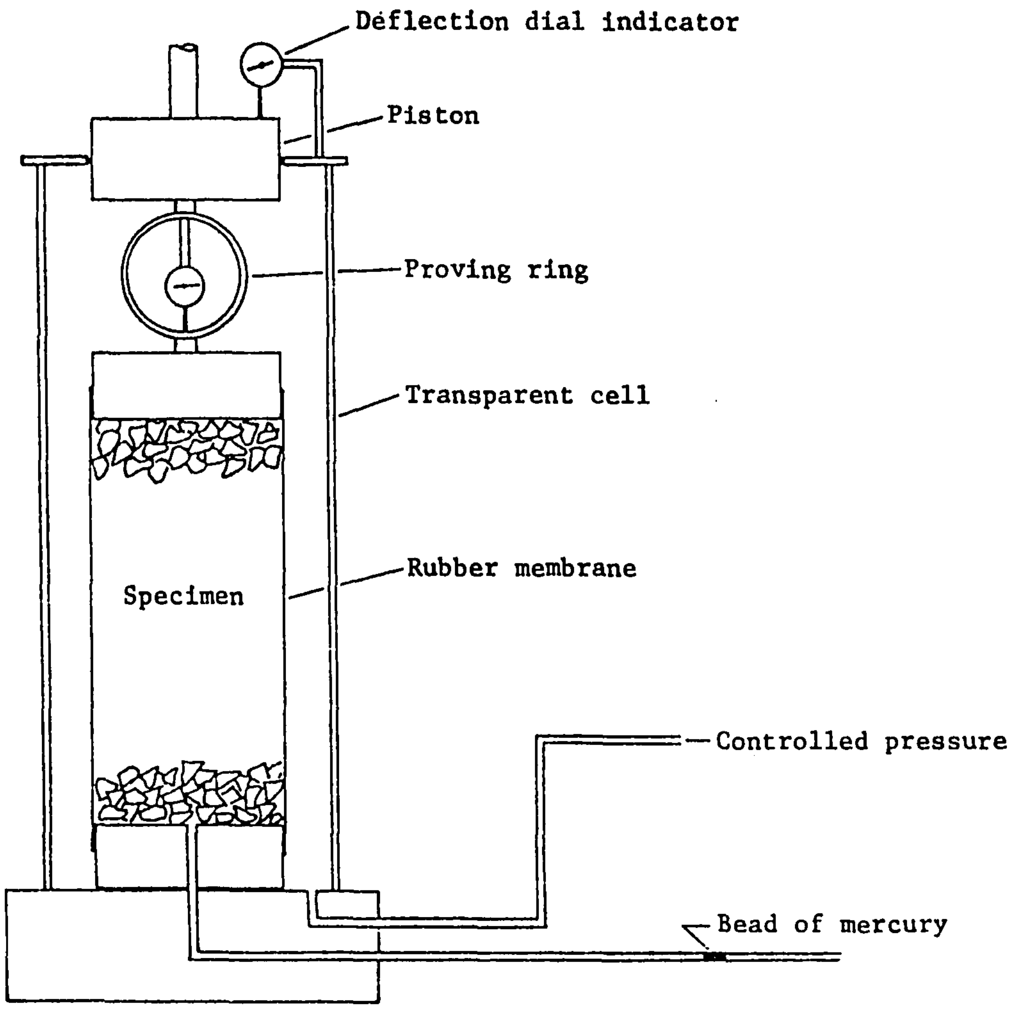

Fig. 2. Schematic diagram of the triaxial test apparatus set up for a constant volume test

comparison to the material being tested, so deflections of the ring do not introduce serious errors in vertical strain measurement and in volume monitoring. These errors may be accounted for by calibrating the volume monitor.

The sample size was set at 305 mm x 152 mm diameter, which

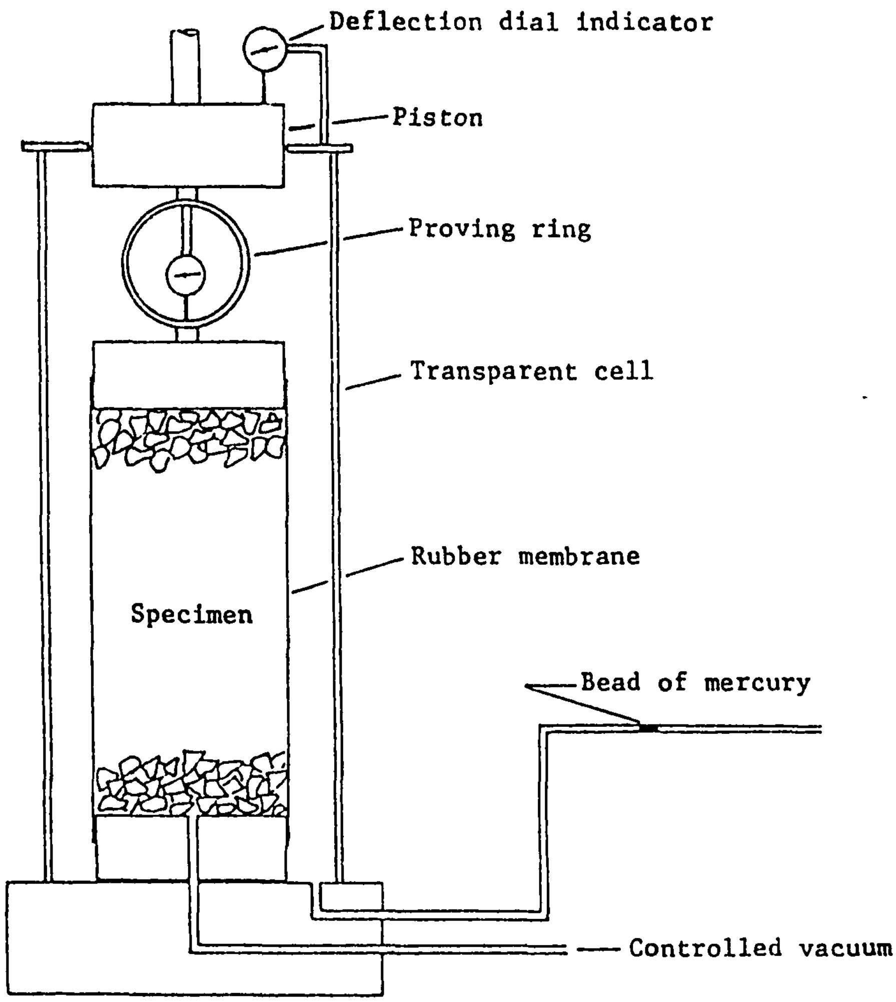

Fig. 3. Schematic diagram of the triaxial test apparatus set up for a zero lateral strain test

corresponds to the largest size of rubber membrane commercially available for soil testing. Further details of the apparatus appear in Appendix B.

Preliminary Test Results

Preliminary tests on maize were undertaken to evaluate the performance of the apparatus, and provide information on which to base a full-scale testing program. Some preliminary results are discussed here.

Standard tests

Five standard tests, with normal stresses between 4 and 40 kPa, were performed on specimens of maize prepared at three initial bulk densities. The Mohr failure envelopes are presented in Fig. 4 where, for each density, the envelope deviates slightly from a straight line and curves downward with increasing normal stress, to produce lower friction angles at higher stresses. The friction angle varies from 30° to 23° in the stress range 4 to 40 kPa.

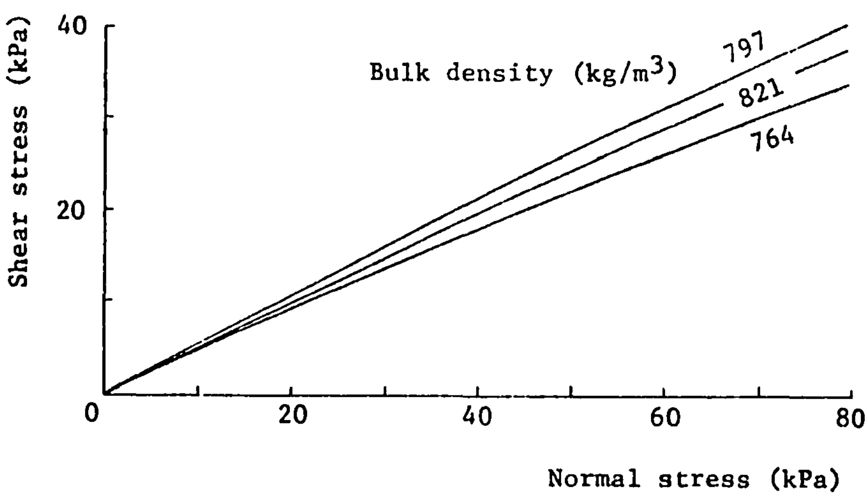

Fig. 4. Mohr failure envelopes for three nominal bulk densities of 15.2% moisture content d.w.b. maize

The relationship between bulk density after isotropic consolidation, friction angle and lateral stress is depicted in Fig. 5. The general trend for lower friction angles at higher lateral stresses is more readily observed. The friction angle increases with increasing bulk density up to a maximum of about 800 kg/m^3, and then decreases at higher bulk densities.

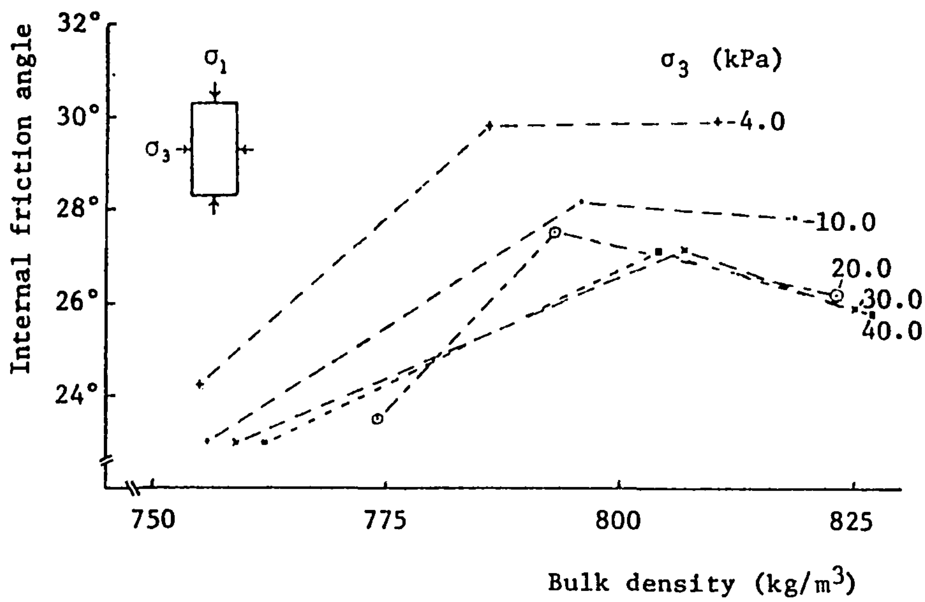

Fig. 5. Internal friction angle vs. bulk density at various confining stresses for 15.2% moisture content d.w.b. maize

Constant volume tests

Constant volume tests on maize required that the lateral stresses be increased above the 70 kPa limit of the testing machine to achieve failure. These results indicate that maize does have a tendency to dilate. Three tests were performed at constant volume up to 2, 4 and 8% axial strain, and then continued as constant lateral stress tests to failure. The Mohr failure envelope for these tests and the stress path, the locus of the tops of the Mohr's circles generated throughout a test, are shown in Fig. 6 for one test.

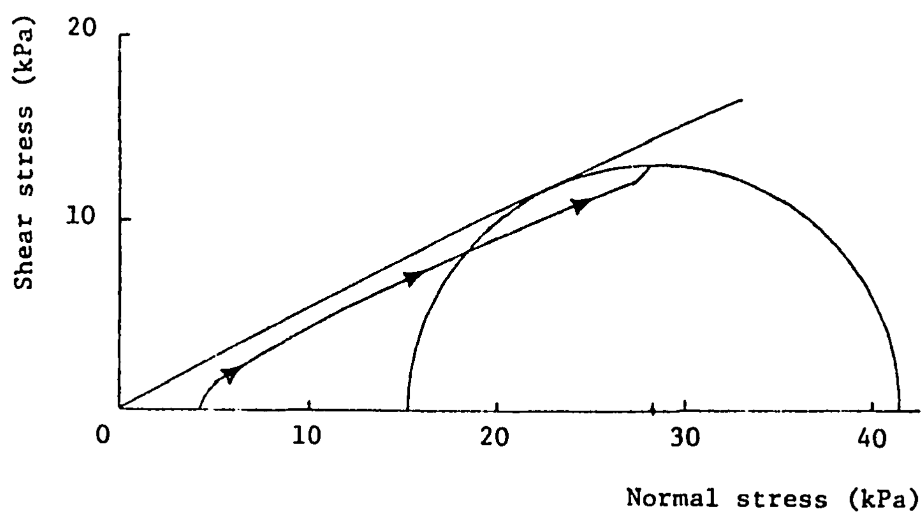

Fig. 6. Mohr failure envelope for partial constant volume tests on 15.2% moisture content d.w.b., 793 kg/m^3 initial bulk density maize. The stress path and Mohr failure circle for the constant volume up to 8% axial strain test are shown

The failure condition was achieved in a constant volume test performed with the same initial bulk density and confining stress, but at a strain rate three times as great as that used in the part constant volume, part standard triaxial tests. The stress-strain relationships for this test and for a nonfailure, constant volume test performed at the lower strain rate are shown in Fig. 7. The increase in lateral stress necessary to maintain constant volume during axial compression is less rapid for the higher strain rate.

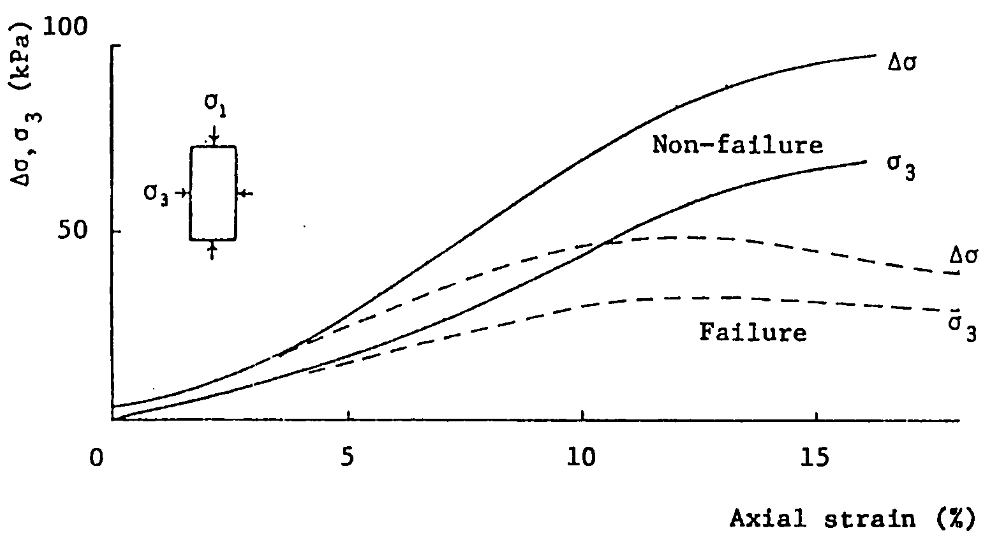

Fig. 7. Lateral stress, σ_3, and deviator stress, $\Delta\sigma = (\sigma_1 - \sigma_3)$, vs. axial strain for failure and nonfailure constant volume tests performed at 1.27%/min and 0.42%/min strain rates, respectively, on 15.2% moisture content d.w.b., 808 kg/m^3 initial bulk density maize

Zero lateral strain tests

Three zero lateral strain tests were performed on maize. In each case, an initial confining stress of 4 kPa was applied, followed by axial compression under zero lateral strain conditions up to 1.0, 2.0 or 3.2% axial strain, followed by axial compression to failure under standard test conditions. The stress-strain relationships for the 3.2% axial strain test are shown in Fig. 8; the Mohr failure envelope and the stress paths for all the tests are shown in Fig. 9.

Discussion and Interpretation of Results

Although a constant angle of internal friction is usually assumed for silo design, the results of the standard triaxial tests show that the Mohr envelope is not linear, but curves downward at higher stress levels. The lower friction angles at higher stresses are interpreted as the result of the deformation of individual particles in the mass. As the stress level is increased, the flattening of particles results in less interlocking, and thus lower friction angles. A similar behavior has been observed in soft, carbonate sands, as contrasted with the linear Mohr envelopes of hard, quartz sands.

Constant volume tests on maize revealed that the grain does dilate when sheared; but at slow strain rates, the failure stresses are beyond the capability of the testing apparatus. More rapid strain rates result in failure at stresses well within the capacity of the machine. In a constant volume test, the lateral stress is continually adjusted to compensate for the tendency for volume increase, thus the only way for failure to occur is by flattening of the individual grains. The observation

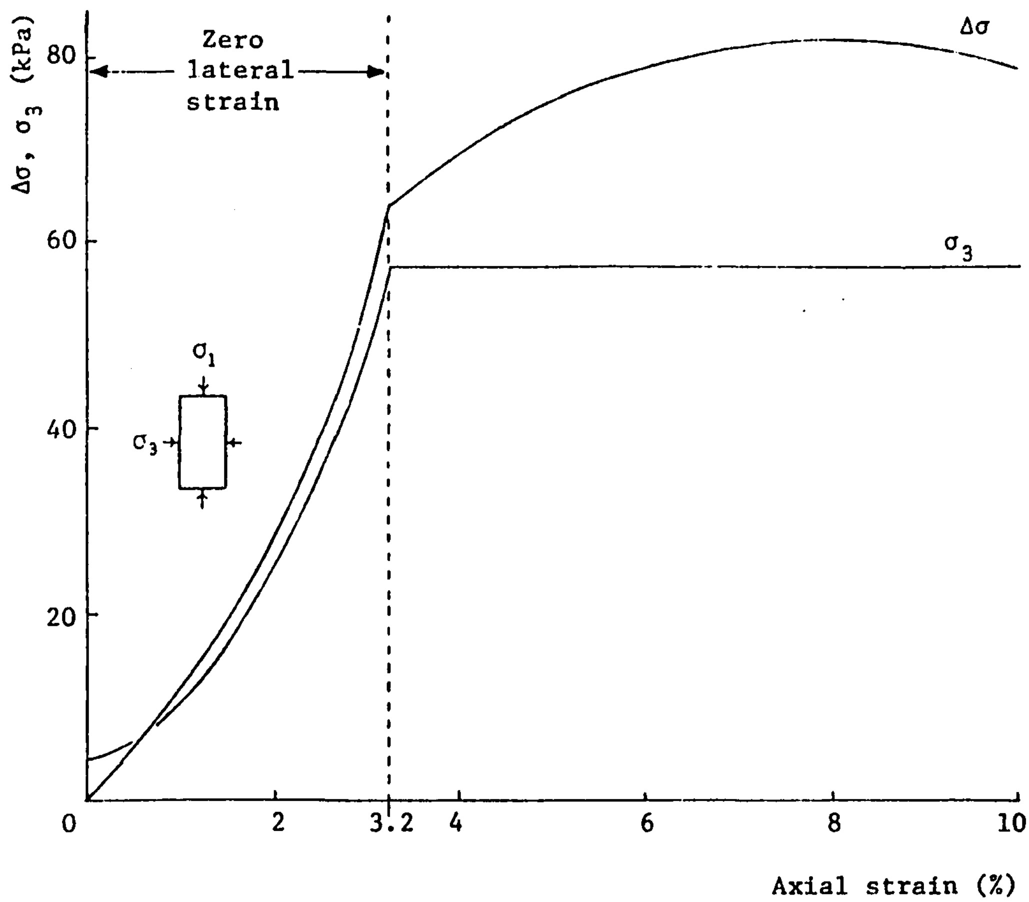

Fig. 8. Lateral stress, σ_3, and deviator stress, $\Delta\sigma = (\sigma_1 - \sigma_3)$, vs. axial strain with zero lateral strain up to 3.2% axial strain for 15.2% moisture content d.w.b., 787 kg/m^3 initial bulk density maize

that the rate of increase of lateral stress to maintain constant volume is slower for the faster strain rate leads to the speculation that deformation characteristics of the particles are dependent upon strain rate, with the particles less compressible at slower strain rates.

The zero lateral strain tests resulted in lateral stress ratios of 0.46 to 0.47. The measured values can be compared with lateral stress

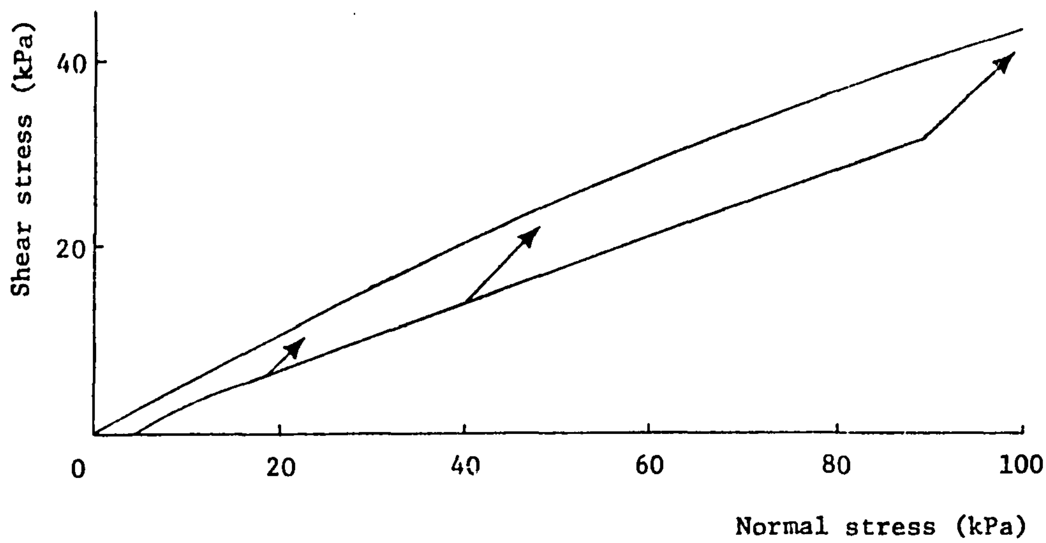

Fig. 9. Mohr failure envelope and stress paths for zero lateral strain up to 1, 2 and 3.2% axial strain tests on 15.2% moisture content, 787 kg/m^3 initial bulk density maize

ratios estimated by Jàky's equation, $k_o = 1 - \sin\phi$, where ϕ is the angle of internal friction of the granular material, and k_o is the lateral stress ratio for zero lateral strain. For the friction angles in this stress range, the lateral stress ratios would be from 0.54 to 0.55. This shows reasonably good agreement, but suggests the possibility that the constant, 1, in Jàky's equation be replaced with a constant of about 0.92 for maize.

Conclusions

The triaxial tests on maize indicate that the grain does dilate during shear, and support the hypothesis (Smith and Lohnes, 1980a) that dilation may be the cause of overpressures in silos. A consideration of both standard and constant volume tests indicates that deformation of individual particles within the granular system may be important in influencing the stress-strain properties of the bulk aggregate. It appears that more attention should be given to the stress-strain characteristics of granular material stored in silos, and that silo design based upon limiting stress analysis may be somewhat unrealistic.

PART III. FRICTIONAL AND STRESS-STRAIN CHARACTERISTICS OF SELECTED AGRICULTURAL GRAINS AS INDICATED BY TRIAXIAL TESTING

Abstract

Maize, wheat, barley and oats were tested in a specially designed triaxial test apparatus. Standard and constant volume tests indicated that stress-strain characteristics are most dependent upon grain packing and stress history, whereas frictional behavior is more dependent upon the geometry and surface roughness of the individual grains. The tendency for dilation increased with decreasing void ratio, regardless of the characteristics of the individual grains.

Introduction

Silo design has employed a limiting stress approach to compute the lateral stresses which the enclosed grain will exert upon the walls of the structure. It is presumed that the grain is at failure and the limiting lateral stresses are a function of the angle of internal friction, ϕ, of the stored material. Although most design manuals report values for ϕ, they recommend that design parameters be determined from tests on the materials which will be stored in the silo (American Concrete Institute, 1977). Several laboratory investigations have related the frictional characteristics of agricultural grains to factors such as bulk density and moisture content (Stewart, 1968; Mohsenin, 1970; Munroe and Moysey, 1974; Moysey et al., 1979; and Lawton and Marchant, 1980).

Recent studies have suggested that laboratory evaluation of stress-strain characteristics may provide insights which will be valuable in determining design stresses in silos. Moysey et al. (1979) presented direct shear data which indicate that wheat dilates during shear. Marchant (1980) has proposed a stress-strain model which relates shear and normal strain increments to shear and normal stress increments, in order to predict the shear dilatance effect. Marchant's model was tested using triaxial data on five agricultural grains, and the results show a good correspondence of predicted and observed strains. Smith and Lohnes (1980 a,b) hypothesized that overpressures which occur when silos are unloaded may result from dilation of grain during shear, and presented triaxial evidence that maize increases in volume when sheared in triaxial testing. This paper describes additional data on the frictional and stress-strain behavior of maize, wheat, barley and oats as measured in a triaxial apparatus.

Testing Apparatus and Methods

A triaxial test apparatus that is used for testing soils is inappropriate for evaluating the mechanistic behavior of agricultural products because the sizes of the individual particles of agricultural grains are much larger than individual soil particles. Also, the stress ranges involved in the storage of agricultural grain are lower than those considered for soils. Consequently, a special apparatus for testing agricultural products was designed for this research.

A schematic of the apparatus is shown in Fig. 1. The apparatus tests specimens nominally 300 mm in height and 150 mm in diameter. For standard triaxial tests in which the confining stress is held constant while the axial stress is increased to failure, the lateral stress is

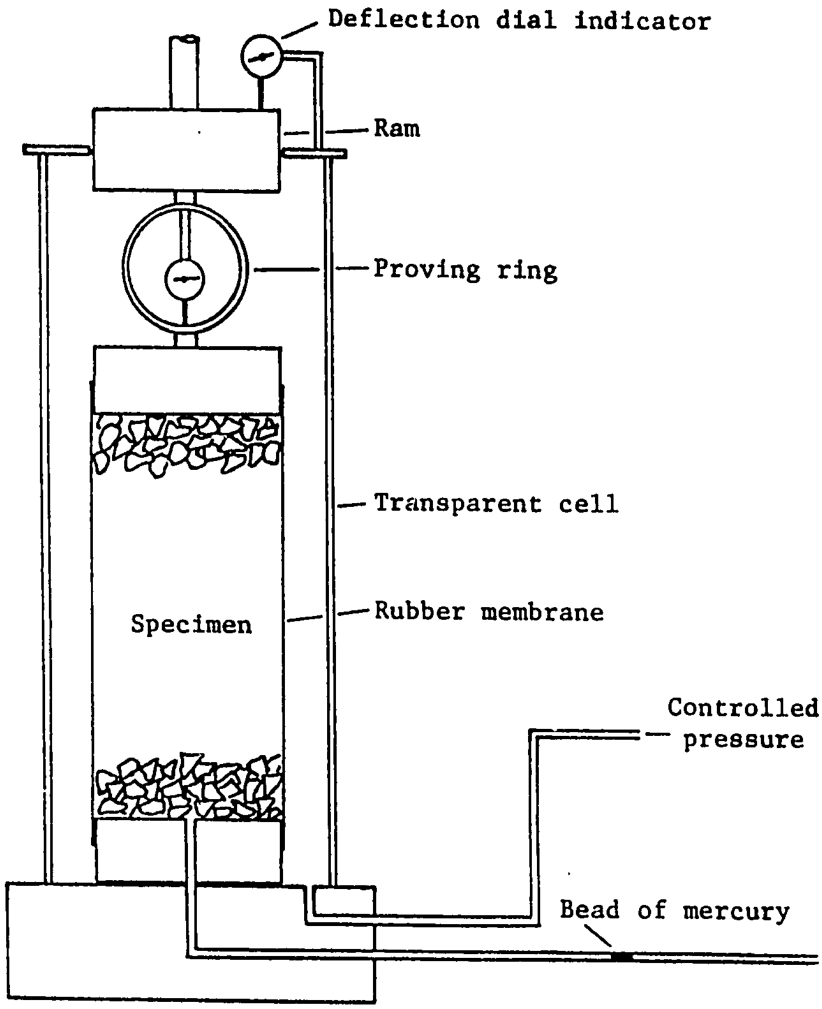

Fig. 1. Schematic diagram of the triaxial test apparatus set up for a constant volume test

applied by increasing the air pressure in the cell and venting the sample pore space to the atmosphere. In constant volume tests, the lateral stress is applied in the same fashion, but adjusted throughout the test to maintain the specimen at constant volume. This is achieved by coupling the specimen pore space to a volume monitor, which consists of a mercury bead in a horizontal capillary tube; any change in specimen pore space will cause air to enter or leave the capillary tube, thereby displacing the mercury bead. Constant volume is achieved by adjusting the confining pressure such that the mercury bead remains stationary in the capillary tube.

A unique feature of this apparatus, which facilitates zero lateral strain tests, is that the ram, which enters the cell to apply the axial stress, has the same cross sectional area as the specimen. The confining stress is applied by partially evacuating the specimen pore space. Zero lateral strain of the specimen is maintained indirectly by connecting the volume monitor to the cell air space, and by adjusting the partial vacuum in the pore space, such that the mercury bead remains stationary. Because the cross sectional area of the entering ram and the shortening specimen are equal, a constant cell air volume creates a condition of zero lateral strain for the specimen. The test apparatus is designed to perform tests at lateral stresses up to 70 kPa. More details of the test apparatus can be found in appendix B.

Test Results

Standard triaxial tests and constant volume tests were performed on maize, wheat, barley and oats at various bulk densities. All grain specimens were at an air dry state, and their moisture contents, and particulate and bulk densities are shown in Table 1. The particulate and bulk density data allow the calculation of void ratios, which are also shown in Table 1.

Table 1. Frictional, bulk and particulate properties of grains studied

Grain	Moisture content % d.w.b.	Particulate density kg/m^3	Bulk density kg/m^3	Friction angle deg	Void ratio	Shape factor
Maize	15.2	1293	821	26.0	0.575	0.49
	15.2	1293	797	27.4	0.622	0.49
	15.2	1293	764	23.2	0.692	0.49
Wheat	10.3	1402	836	24.3	0.677	0.60
	10.3	1402	898	25.8	0.560	0.60
Barley	13.3	1398	691	28.5	1.023	0.49
	13.3	1398	752	30.9	0.858	0.49
Oats	12.2	1349	565	31.1	1.375	0.37

It is recognized that several variables in addition to moisture content and bulk density (or void ratio) influence the strength and stress-strain characteristics of particulate systems. Two factors that seem especially important are the surface roughness of the individual grains and grain shape. Sample grains were studied with a scanning electron microscope at magnifications of 50 x and 300 x. No attempt was made to describe quantitatively the surface roughness, but qualitatively the oats exhibited the roughest surface texture of the four grains, with barley the second most rough. Wheat and maize are similar in surface texture, and considerably smoother than barley. Micrographs of the grains can be found in Appendix C.

The shape of individual grains was characterized by a shape factor. Representative grains were measured with a micrometer along their longest, shortest and intermediate axes. The shape factor is the shortest dimension divided by the square root of the product of the longest and intermediate dimensions. Thus, an equidimensional particle has a shape factor of one; and the more elongate the particle, the smaller the shape factor. The average shape factor for each grain is shown in Table 1, where it can be seen that wheat has the most nearly equidimensional grains, and oats the most elongate. Both barley and maize have grains intermediate in shape.

The friction angles reported in Table 1 assume a linear relationship between normal and shearing stress; however this and previous work indicate that the failure envelope for most grains has a concave down curvature (Moysey et al., 1979; Smith and Lohnes, 1980b). It is interesting that if all the frictional data are compared, oats (with the highest void ratio) has a high friction angle, whereas wheat and maize (with low void ratios) have low friction angles. The observation that high void ratio materials have higher friction angles than low void ratio materials is anomalous, because usually lower void ratio materials with more dense packing of particles would result in higher friction angles.

Constant volume tests were conducted to assess the tendency for dilation in the four materials. In the constant volume test, the lateral stress is adjusted throughout the axial loading of the specimen, such that no volume change is allowed as the sample is loaded axially. If the specimen has a tendency to decrease in volume during loading, the lateral stress must be decreased. If the specimen has a tendency to dilate or expand during the axial loading, the lateral stress must be increased to counteract the tendency for volume increase. Constant volume tests were conducted on maize at three bulk densities, on wheat and barley at two bulk densities each, and on oats at one bulk density. It was observed that at a strain rate of 0.42%/min, none of the specimens failed, as indicated by an axial stress decrease after reaching some maximum value. It was further observed that if specimens of maize were loaded at more rapid rates, a distinct maximum stress was attained. This behavior is attributed to the fact that in order for a particulate system to fail under constant volume, there must be a deformation or shearing of the individual particles, and the reason for failure at faster strain rates is that the individual particles are less compressible at lower strain rates (Smith and Lohnes, 1980 b). The magnitude of the lateral stress at failure is a measure of the tendency of the material to dilate, because the greater the tendency to expand during axial loading, the higher the confining stress must be to counteract this tendency.

In order to evaluate the tendency for dilation among the four grains tested in this study, an arbitrary failure criterion for the constant volume tests was adopted. Failure was assumed to occur at the strain at which failure occurred in the corresponding standard triaxial test. Table 2 shows the failure strains, void ratios, and minor principal stresses at which failure occurred for each of the constant volume tests. The lateral stress in the standard tests and the initial lateral stress in the constant volume tests were 4.00 kPa.

Table 2. Results of constant volume tests

Grain	Void ratio	Failure strain %	Lateral stress at failure (kPa) Strain rate (%/min) 0.42	1.52	1.67
Maize	0.692	12.36	--	5.4	--
	0.622	6.72	13.7	20.6	--
	0.575	8.33	34.8	--	--
Wheat	0.677	10.58	10.0	8.3	9.3
	0.560	12.24	58.5	45.1	47.9
Barley	1.023	14.59	2.8	3.6	3.3
	0.858	11.77	13.3	13.3	15.5
Oats	1.375	15.07	2.1	1.7	2.1

Discussion

The frictional characteristic of a particulate system is the result of the frictional resistance of one material sliding against another in combination with an interlocking effect. In soils, the interlocking is much more important than the frictional resistance between individual particles, with the result that a decrease in void ratio results in higher values of ϕ. The data in Table 1 are contradictory to this generalization, because barley and oats with the highest void ratios also have the highest friction angles. The data in Table 1 indicate that grain shape is also an important factor, because wheat, the most equidimensional material, has the lowest values of ϕ, whereas oats, with the most elongate particles, has much higher values of ϕ. Grain shape does not account for all the variation between the grains tested, because both barley and maize have essentially the same shape factor, but the friction angle of barley is higher than that of maize. This may be explained by the electron micrographs, which indicate that the surface roughness of the barley is greater than that of the maize. Also, if the values of ϕ for both maize and barley are compared, it can be seen that the higher void ratio barley has higher values of ϕ. This observation suggests the importance of surface roughness on the frictional resistance of agricultural grain.

The constant volume tests indicate that low void ratio materials have the greatest tendency to dilate, regardless of the characteristics of the individual grains. The constant volume tests also show a sensitivity to strain rate, but no clear trend is apparent. The use of void ratio rather than bulk density to characterize the packing of the granular material removes the influence of particulate density, and as such is a better property to compare the effect of packing when several grains are considered. If the void ratio of each of the grains is plotted versus the lateral stress at the arbitrary failure rate, a rough correlation exists, with the lower void ratio materials exhibiting higher stresses and thus a greater tendency to dilate. The relationship between void ratio and lateral stress at failure is shown in Fig. 2. These data indicate that when agricultural grain is loaded in axial compression, low void ratio materials tend to dilate more than high void ratio materials. This behavior is consistent with the behavior of noncohesive soils.

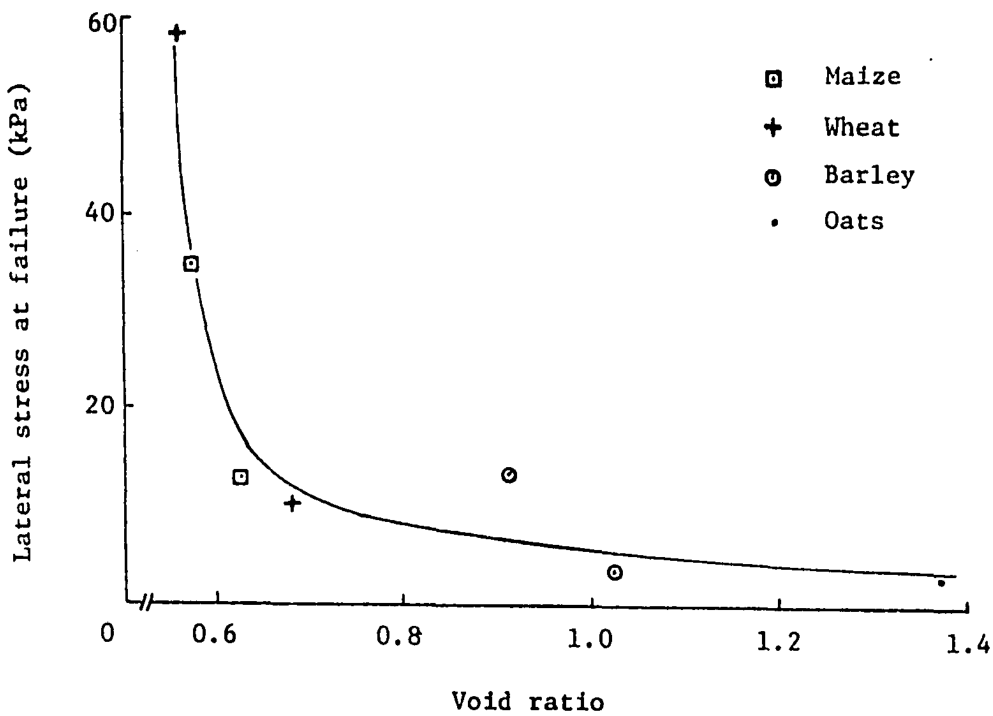

Fig. 2. Lateral stress at failure vs. void ratio. Constant volume tests on four grains

Conclusions

The results of this investigation indicate that the special low pressure triaxial apparatus gives values for ϕ which fall within the range of published values for maize, wheat, barley and oats.

The frictional characteristics of the grain vary depending upon packing, particle shape, and surface roughness, but an analysis of the independent variables suggests that the frictional behavior of the grain may depend more upon shape and roughness than upon packing. Rougher and more elongate particles result in higher values of ϕ.

The tendency for dilation during axial compression is greater in grains with low void ratios. These tests suggest that stress-strain characteristics of agricultural grains are more dependent upon packing and stress history of the bulk material, whereas frictional behavior is more dependent upon the properties of the individual grains which comprise the bulk material.

PART IV. FRICTIONAL PROPERTIES AND STRESS-STRAIN RELATIONSHIPS FOR USE IN THE FINITE ELEMENT ANALYSIS OF GRAIN SILOS

Introduction

With the advent of the modern digital computer the analysis of structures by iteration techniques, such as finite element analysis, became feasible. The finite element method consists of dividing a structure and, in the case of a silo, its contents into many small elements which are interconnected by nodes. Equilibrium equations may be written for each element in terms of nodal displacements, the element stiffness matrix, and the applied nodal stresses. The element stiffness matrix depends upon the geometry and the material properties of the element.

Although finite element models for silo design have been developed, little attention has been paid to evaluating the relevant properties of the stored material (Fanous, 1980). This work evaluated some of those properties for four agricultural grains.

In the conventional stress-strain analysis of isotropic materials, only two independent coefficients of elasticity exist (Dally and Riley, 1978). For linear elastic materials, these coefficients are constant and may be used directly in the stiffness matrix. For nonlinear elastic materials, stepwise linear or functional relationships may be used. Conventional stress-strain analysis, with only two independent coefficients, attributes normal strains to normal stresses alone, and shear strains to shear stresses alone. Marchant (1980) points out that the lack of cross-coupling between shear stresses and normal strains may produce values of Poisson's ratio greater than 0.5 for granular materials, due to volume increases (i.e. dilation) due to shearing. A material with a Poisson's ratio greater than 0.5 would have a negative bulk modulus, and anomalous volume increases would be predicted under increasing hydrostatic stress conditions. Marchant proposed an elastic model for granular materials, with four independent coefficients of elasticity which provided the necessary cross-coupling. While the importance of the shear dilatance effect is recognized (Smith and Lohnes, 1980 a,b, 1981), this dissertation evaluates normal stress-uniaxial normal strain relationships which are not dependent upon shear strains. These relationships are suitable for use in most finite element programs currently developed for predicting stresses and strains in grain silos.

Material Properties

Stress-strain characteristics

The stress-strain modulus of grain is stress dependent, as is shown in Fig. 1. Konder and Zelasko (1963) have shown that stress-strain curves for cohesionless materials may be approximated by hyperbolas of the form:

$$\Delta\sigma = \frac{\varepsilon_a}{\frac{1}{E_i} + \frac{\varepsilon_a}{\Delta\sigma_{ult}}} \qquad [1]$$

where $\Delta\sigma = \sigma_1 - \sigma_3$ (where σ_1 and σ_3 are major and minor principal stresses, respectively)

ε_a = axial strain

E_i = initial tangent modulus of elasticity

$\Delta\sigma_{ult}$ = the asymptotic maximum value of $\Delta\sigma$, termed the asymptotic compressive strength (see Fig. 1)

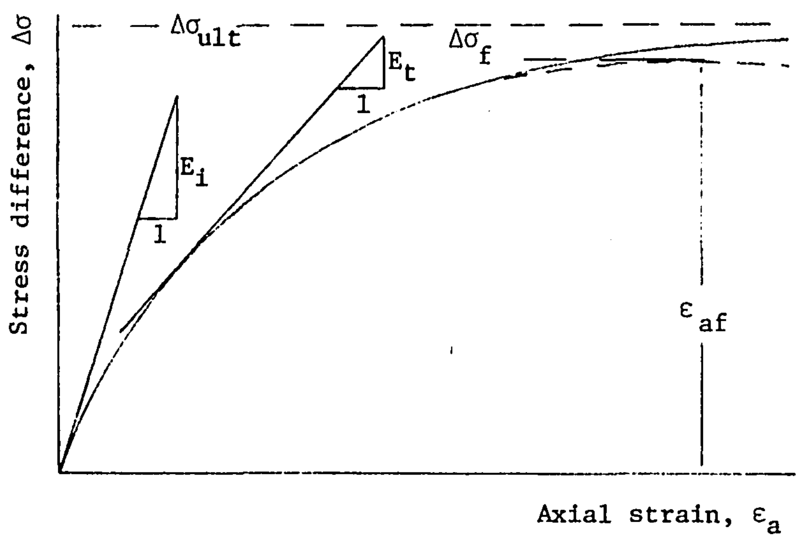

Fig. 1. Tangent moduli (E_i and E_t), compressive strengths ($\Delta\sigma_{ult}$ and $\Delta\sigma_f$), and failure strain (ε_{af}) for an hyperbolic stress-strain material

Rearranging equation 1 results in:

$$\frac{\varepsilon_a}{\Delta\sigma} = \frac{1}{E_i} + \frac{\varepsilon_a}{\Delta\sigma_{ult}} \qquad [2]$$

which is linear with variables $\frac{\varepsilon_a}{\Delta\sigma}$ and ε_a.

Janbu (1963) has shown that the relationship between the initial tangent modulus and the minor principal stress may be expressed by:

$$E_i = k_h \, Pa \left(\frac{\sigma_3}{Pa}\right)^n \qquad [3]$$

where k_h and n are empirical, dimensionless constants

Pa = atmospheric pressure expressed in the same units as E_i and σ_3

The inclusion of atmospheric pressure is necessary if k_h is to be dimensionless; the value of n, however, is necessarily dimensionless and numerically independent of the system of units. SI units are used in this work, so equation 3 was modified to:

$$E_i = k\,P^{(n-1)}\,\sigma_3{}^n \qquad [4]$$

where $P = \dfrac{\text{unit stress in desired system}}{\text{unit stress in SI system (1 kPa)}}$

Equation 4 is linear in logarithmic space with variables E_i and σ_3. If the tangent modulus, E_t (see Fig. 1), is defined as:

$$E_t = \frac{\delta(\Delta\sigma)}{\delta\,\varepsilon_a} \qquad [5]$$

then rearranging equation 2 and differentiating:

$$E_t = \frac{1}{E_i\left[\frac{1}{E_i} + \frac{\varepsilon_a}{\Delta\sigma_{ult}}\right]^2} \qquad [6]$$

Solving equation 1 for ε_a and substituting in equation 6:

$$E_t = E_i\left[1 - \frac{\Delta\sigma}{\Delta\sigma_{ult}}\right]^2 \qquad [7]$$

As the result of the triaxial tests on the selected agricultural grains, the relationship between the major and minor stresses at failure (σ_{1f} and σ_{3f}, respectively) was best expressed by:

$$\sigma_{1f} = b\,P^{(c-1)}\,\sigma_{3f}{}^c \qquad [8]$$

where b and c are empirical dimensionless constants

Equation 8 is linear in log space with variables σ_{1f} and σ_{3f}. Thus:

$$\Delta\sigma_f = \sigma_{1f} - \sigma_{3f} = b\,P^{(c-1)}\,\sigma_{3f}{}^c - \sigma_{3f} \qquad [9]$$

where $\Delta\sigma_f$ = the empirical, maximum value of $\Delta\sigma$, termed the experimental compressive strength (see Fig. 1)

The value of $\Delta\sigma_f$ may be less than $\Delta\sigma_{ult}$, i.e. the experimental compressive strength may be less than the asymptotic compressive strength.

Letting $\frac{\Delta\sigma_f}{\Delta\sigma_{ult}} = R$ and substituting equations 4 and 9 in equation 7:

$$E_t = [1 - \frac{\Delta\sigma R}{b P^{(c-1)} \sigma_{3f}^{c} - \sigma_{3f}}]^2 k P^{(n-1)} \sigma_3^{n} \qquad [10]$$

In the SI system of units P is unity and the general equation reduces to:

$$E_t = [1 - \frac{\Delta\sigma R}{b \sigma_{3f}^{c} - \sigma_{3f}}]^2 k \sigma_3^{n} \qquad [11]$$

which is a restricted equation.

The constants in equation 11 were evaluated by triaxial testing and by least squares fitting of equations 2, 4 and 8.

Frictional properties

A nonlinear relationship between the major and minor principal stresses at failure results in a nonlinear Mohr failure envelope. In triaxial testing the normal and shear stresses on the failure plane at failure are not measured but, if a linear relationship between the major and minor stresses is assumed, they may be found by simple geometric manipulation. No closed solution could be found for the Mohr failure envelope when the major and minor stresses at failure are not linearly related, so a numerical approximation technique was developed.

The desired Mohr failure envelope is tangent to all the Mohr failure circles. The point of tangency for each failure circle is not known, but may be approximated by the point which is midway between the points of tangency of straight lines drawn tangent to the neighboring failure circles, as shown in Fig. 2. The latter points of tangency may be found by geometric manipulation of the coordinates of the tops of the failure circles. Details of the manipulation appear in appendix D. For the selected agricultural grains the relationship between the shear stress, τ_f, and the normal stress, σ_f, on the failure plane at failure was best expressed by:

$$\tau_f = a\,P^{(m-1)}\,\sigma_f^{\ m} \qquad [12]$$

where a and m are empirical, dimensionless constants

In the SI system of units the general equation reduces to:

$$\tau_f = a\,\sigma_f^{\ m} \qquad [13]$$

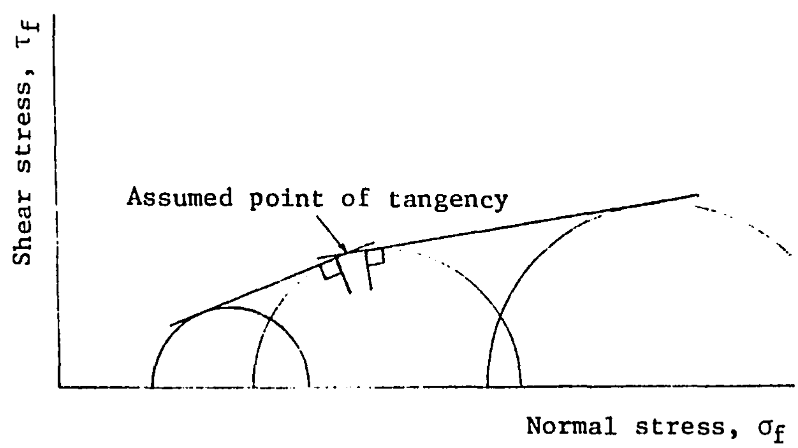

Fig. 2. Location of the assumed point of tangency of a nonlinear Mohr failure envelope to a failure circle

Results

The stress-strain curves for a typical grain are shown in Fig. 3, and the experimental ultimate shear strengths and failure strains for all the grains are given in Tables 1a - 1d. The values of the constants in equation 2, and of R (the ratio of experimental to asymptotic ultimate shear strengths) for all the grains are given in Tables 2a - 2d. The values of the constants in equations 10 and 11 for all the grains are given in Table 3.

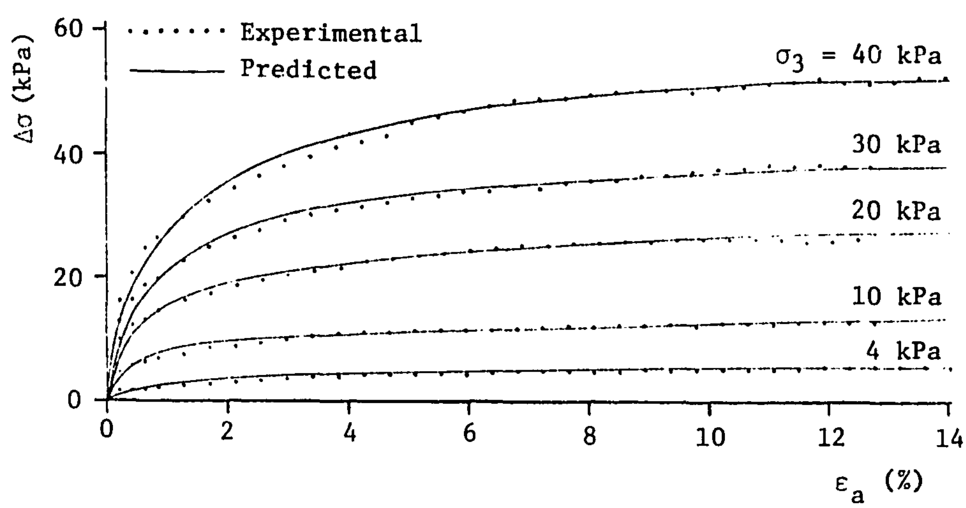

Fig. 3. Normal stress difference ($\Delta\sigma$) vs. axial strain (ε_a) at five lateral stresses (σ_3) for maize at 764 kg/m^3 bulk density, 15.2% moisture content d.w.b.

Table 1a. Experimental ultimate shear strengths ($\Delta\sigma_f$) and axial failure strains (ε_{af}), for maize at 15.2% moisture content d.w.b.

	764 kg/m^3		797 kg/m^3		821 kg/m^3	
σ_3 kPa	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %
4	5.8	12.8	7.9	4.0	8.0	8.3
10	12.9	11.9	17.9	4.7	17.4	6.8
20	26.4	12.5	34.6	4.6	31.7	5.1
30	38.6	11.9	51.2	6.5	46.6	8.9
40	52.7	13.6	68.0	6.0	61.8	9.7

Table 1b. Experimental ultimate shear strengths ($\Delta\sigma_f$) and axial failure strains (ε_{af}), for wheat at 10.3% moisture content d.w.b.

	836 kg/m^3		862 kg/m^3		898 kg/m^3	
σ_3 kPa	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %
4	6.2	10.6	6.9	3.0	6.6	12.2
10	15.0	13.9	18.3	4.2	15.2	12.7
20	28.7	10.1	34.4	3.0	31.9	4.6
30	41.4	11.9	50.1	5.1	46.8	7.3
40	55.6	11.1	67.4	4.7	60.6	8.1

Table 1c. Experimental ultimate shear strengths ($\Delta\sigma_f$) and axial failure strains (ε_{af}), for barley at 13.3% moisture content d.w.b.

	691 kg/m^3		715 kg/m^3		752 kg/m^3	
σ_3 kPa	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %
4	7.9	14.6	10.2	6.5	8.7	11.8
10	19.3	16.3	22.6	9.5	23.0	11.6
20	37.1	15.2	44.5	8.1	43.3	12.9
30	54.7	13.8	70.6	9.5	62.8	14.9
40	72.5	15.8	91.7	9.5	83.6	15.6

Table 1d. Experimental ultimate shear strenghts ($\Delta\sigma_f$) and axial failure strains (ε_{af}), for oats at 12.2% moisture content d.w.b.

	565 kg/m^3		588 kg/m^3		597 kg/m^3	
σ_3 kPa	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %	$\Delta\sigma_f$ kPa	ε_{af} %
4	10.1	15.1	12.5	16.8	12.2	10.8
10	24.3	17.1	26.6	17.1	24.6	13.1
20	46.9	17.8	46.7	13.2	43.3	17.5
30	64.9	17.1	68.2	17.3	67.4	17.7
40	82.7	17.8	91.0	15.6	92.6	17.9

Table 2a. Initial tangent moduli (E_i), asymptotic ultimate shear strengths ($\Delta\sigma_{ult}$), and ratios of asymptotic to experimental ultimate shear strengths (R), for maize at 15.2% moisture content d.w.b.

	764 kg/m^3			797 kg/m^3			821 kg/m^3		
σ_3 kPa	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R
4	327	6.1	0.941	2857	8.0	0.839	1532	8.2	0.976
10	1446	13.6	0.947	6274	18.5	0.968	3035	18.7	0.934
20	3056	28.3	0.931	8937	37.3	0.927	6978	34.3	0.925
30	3793	41.2	0.936	11501	54.5	0.938	5539	50.8	0.917
40	4930	56.5	0.933	13546	73.1	0.930	8684	66.8	0.924
	Mean R = 0.938			Mean R = 0.920			Mean R = 0.935		
	$E_i = 78.4\sigma_3^{1.163}$			$E_i = 1218\sigma_3^{0.663}$			$E_i = 578\sigma_3^{0.732}$		

Table 2b. Initial tangent moduli (E_i), asymptotic ultimate shear strenghts ($\Delta\sigma_{ult}$), and ratios of asymptotic to experimental ultimate shear strengths (R), for wheat at 10.3% moisture content d.w.b.

	836 kg/m^3			862 kg/m^3			898 kg/m^3		
σ_3 kPa	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R
4	548	6.5	0.944	6115	7.1	0.978	1554	6.2	1.057
10	2241	15.2	0.982	14684	18.8	0.976	3677	15.7	0.969
20	6551	29.6	0.969	19220	36.2	0.950	7689	33.9	0.939
30	7034	43.1	0.959	27752	52.0	0.964	12128	49.1	0.952
40	12970	57.6	0.965	25451	71.5	0.943	12291	63.8	0.950
	Mean R = 0.964			Mean R = 0.962			Mean R = 0.973		
	$E_i = 95.3\sigma_3^{1.332}$			$E_i = 2836\sigma_3^{0.639}$			$E_i = 423\sigma_3^{0.950}$		

Table 2c. Initial tangent moduli (E_i), asyptotic ultimate shear strengths ($\Delta\sigma_{ult}$), and ratios of asymptotic to experimental ultimate shear strengths (R), for barley at 13.3% moisture content d.w.b.

	691 kg/m^3			715 kg/m^3			752 kg/m^3		
σ_3 kPa	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R
4	323	8.9	0.887	1261	11.4	0.894	665	9.5	0.915
10	860	21.8	0.886	2402	25.0	0.904	1668	25.7	0.895
20	1740	42.6	0.872	3633	52.8	0.843	2679	49.2	0.880
30	2732	63.4	0.862	4907	83.3	0.848	3550	71.2	0.882
40	3360	84.2	0.860	5519	111.2	0.824	4256	95.7	0.875
	Mean R = 0.873			Mean R = 0.863			Mean R = 0.889		
	$E_i = 78.8\sigma_3^{1.030}$			$E_i = 524\sigma_3^{0.648}$			$E_i = 237\sigma_3^{0.799}$		

Table 2d. Initial tangent moduli (E_i), asymptotic ultimate shear strengths ($\Delta\sigma_{ult}$), and ratios of asymptotic to experimental ultimate shear strengths (R), for oats at 12.2% moisture content d.w.b.

	565 kg/m^3			588 kg/m^3			597 kg/m^3		
σ_3 kPa	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R	E_i kPa	$\Delta\sigma_{ult}$ kPa	R
4	323	12.1	0.835	537	14.0	0.890	722	14.4	0.849
10	597	31.2	0.779	836	32.2	0.827	893	31.0	0.791
20	917	65.9	0.711	1502	61.9	0.754	1187	55.3	0.783
30	1456	86.8	0.748	1709	90.1	0.757	1517	89.1	0.756
40	1561	117.1	0.706	2090	126.5	0.719	1832	126.9	0.729
	Mean R = 0.756			Mean R = 0.789			Mean R = 0.782		
	$E_i = 119\sigma_3^{0.706}$			$E_i = 227\sigma_3^{0.602}$			$E_i = 385\sigma_3^{0.401}$		

Table 3. Stress-strain constants in equations 10 and 11 for four grains

Grain	Bulk density kg/m^3	R	b	c	k	n
Maize	764	0.938	2.40	0.987	78	1.163
	797	0.920	3.12	0.958	1218	0.663
	821	0.935	3.27	0.927	578	0.732
Wheat	836	0.964	2.65	0.971	95	1.332
	862	0.962	2.81	0.987	2836	0.639
	898	0.973	2.67	0.985	423	0.950
Barley	691	0.873	3.10	0.973	79	1.030
	715	0.863	3.56	0.978	524	0.648
	752	0.889	3.33	0.982	237	0.799
Oats	565	0.756	3.87	0.942	119	0.706
	588	0.789	4.45	0.906	227	0.602
	597	0.782	4.70	0.895	385	0.401

The Mohr failure envelopes for a typical grain at three bulk densities are shown in Fig. 4, and the numerical values of the constants in equations 12 and 13 for all the grains are given in Table 4. Least squares fitting of the constants in equations 8 and 12 showed that n and m were, in all cases, significantly less than unity (α = 0.05), resulting in Mohr failure envelopes which are concave downwards. To facilitate the comparison between grains, however, the more familiar constant internal angles of friction, ϕ, assuming linear Mohr failure envelopes, have been included in Table 4.

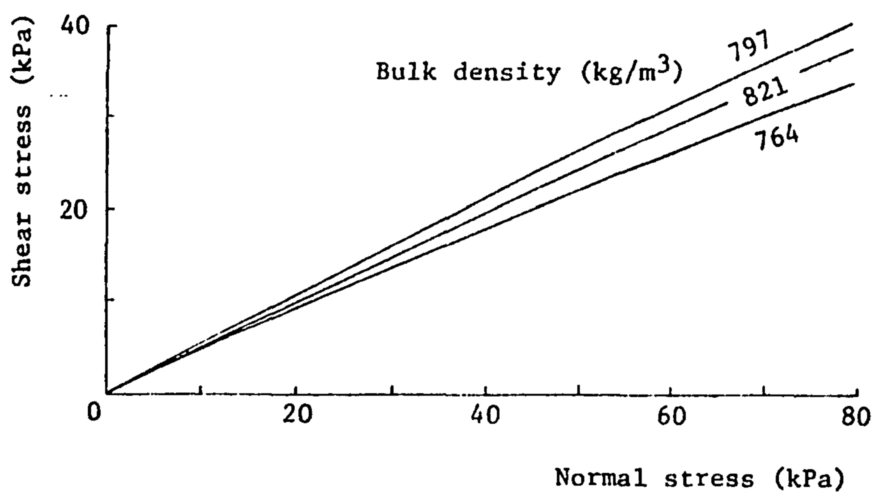

Fig. 4. Mohr failure envelopes for three nominal bulk densities of 15.2% moisture content d.w.b. maize

Table 4. Friction constants in equations 12 and 13, and internal angles of friction, ϕ, for four grains

Grain	Bulk density kg/m^3	Void ratio	a	m	ϕ deg
Maize	764	0.692	0.455	0.983	23.2
	797	0.622	0.612	0.955	27.4
	821	0.575	0.657	0.920	26.0
Wheat	836	0.677	0.515	0.965	24.3
	862	0.626	0.543	0.987	27.2
	898	0.560	0.515	0.983	25.8
Barley	691	1.023	0.605	0.972	28.5
	715	0.955	0.683	0.980	32.5
	752	0.858	0.642	0.983	30.9
Oats	565	1.375	0.753	0.945	31.2
	588	1.294	0.858	0.915	32.2
	597	1.260	0.908	0.905	32.3

Discussion

The experimental and predicted stress-strain curves shown in Fig. 4 are in reasonable agreement, which validates the assumptions of hyperbolic stress-strain relationships, and the linearity in logarithmic space of the initial tangent modulus-minor principal stress relationship. The value of R (see Tables 2a - 2d) does not appear to depend on the minor principal stress, so mean values were assumed. This assumption is reasonable, as the predicted and experimental shear strengths at the experimental failure strains are in close agreement.

Although each grain was tested at only three bulk densities, the parameters of maize, wheat and barley show similar trends as bulk density changes. The friction angle and the initial tangent moduli for each grain are highest, and the failure strain and the exponent, n, are lowest, at the medium bulk densities (Tables 1a - 1d, 2a - 2d, and 4).

The parameters for oats do not follow the above trends, since highest and lowest values occur at the high bulk density. An explanation for this may lie in the range of bulk density investigated. By removing the influence of particulate density, void ratios may be used to make comparisons between grains, as explained by Smith and Lohnes (1981). For oats, the void ratios ranged from 1.260 to 1.375, whereas for maize, wheat and barley they ranged from 0.575 to 0.692, from 0.560 to 0.667, and from 0.858 to 1.023, respectively (Table 4). Thus, the void ratios for the oats were much higher than for the other grains, even though the samples were prepared in a similar fashion. If the oats had been prepared at lower void ratios (i.e. higher bulk densities), then the parameters may have reflected the general trends.

The following analogy to a pile of bricks is offered as an explanation of the general trends. If bricks are placed randomly in a heap in a very open structure (i.e. a high void ratio), then the internal friction angle, which is composed of sliding friction and interlocking, will be relatively low, due to the low degree of interlocking. Applied stresses will shear the heap easily, resulting in a low initial tangent modulus, and full shear strength will only be developed after considerable strain has taken place. If the bricks are now placed randomly, but in a more close structure (i.e. an intermediate void ratio), the friction angle will be relatively high, due to the high degree of interlocking. Applied stresses will not shear the heap so easily, resulting in a high initial tangent modulus. Full shear strength will develop after relatively little strain has taken place, since further strain decreases the interlocking. If a lower void ratio is required, the bricks must be placed with some preferred orientation, rather than at random. Layering of the bricks to reduce the void ratio also reduces the interlocking, causing a return to a lower friction angle, a lower initial tangent modulus, and a higher failure strain.

The general trends are not consistent with classical soil mechanics. This may be due to the uniformity of particle size in bulk grain, since, drawing upon the brick analogy, filling of the intersticial spaces by fine material may prevent preferred orientation from reducing further the already lower void ratio.

At low bulk densities, the exponent, n, is greater than unity, indicating greater than proportional increases in the tangent moduli with increasing minor principal stresses. This may be due to the greater reductions in void ratios, due to consolidation, which are possible at low bulk densities. At the intermediate and high bulk densities, little reductions in void ratios, and hence increases in interlocking, are possible and thus n is low.

The tangent modulus is an important component in the stiffness matrix, and should be evaluated for each material to be stored. The results show that the function for the tangent modulus depends not only on the type of grain, but also on its bulk density. Different filling procedures and degrees of vibration were used to produce the different bulk densities in the triaxial test specimens. The effects of filling procedures and vibration on bulk density need to be evaluated for the prototype structure, in order to determine the appropriate function for the tangent modulus.

Qualitatively the void ratios obtained for each grain may be explained in terms of the surface roughness of the particles. Oats are considerably rougher than barley, which is rougher than wheat or maize (see appendix C), and this order is reflected in the void ratios (Table 4), the rougher grains having higher void ratios.

The triaxial tests reported in this work have yielded data on the compression loading tangent moduli, which may be used in the stress analysis of the loading and storing of grain in a structure. Tangent moduli from extension unloading tests are required for the stress analysis of the unloading process. An electromagnetic tension coupling has been fitted to the triaxial apparatus, to allow such tests to be performed. The volumetric strains, which conventionally would be used to calculate Poisson's ratio, were not measured in most of the tests, but a volumeter has been added to the apparatus, to enable volume change measurements to be made. Details of the tension coupling and volumeter appear in Appendix B.

Most published data on the triaxial testing of particulate materials are for soils, which are generally more complex materials than grain. Grain is of uniform, although complex, shape, and is well sorted in comparison to soils, other than those of eolian and alluvial origin. Cohesion and pore water pressure, which are important parameters in many geotechnical problems, are also absent in dry, large particled, granular systems. Triaxial testing is time consuming and expensive, and is normally only used to aid in the design of specific, large construction projects, since the complex nature of soil systems has so far defeated attempts to relate bulk stress-strain behaviors to particulate characteristics. Bulk grain has not been subjected to extensive triaxial testing in conjunction with particle characterization, but, because of its less complex nature, it may be possible to predict its bulk behavior from its particle characteristics. It has already been shown qualitatively (Smith and Lohnes, 1981) that bulk frictional characteristics are more dependent upon surface roughness than interlocking or stress history. In a similar manner, bulk stress-strain relationships may be related to particle stress-strain relationships, shape, surface roughness and so on, and simple bulk properties such as void ratio. If this is so, the data from the detailed triaxial testing of relatively few granular materials may be extrapolated to cover a wide range of materials, for which the pertinent particle characteristics are known.

Conclusions

The results of this work indicate that the triaxial test is an effective means of evaluating the stress-strain relationships of agricultural grains, and that the less complex nature of these dry, granular materials leads to more consistent results than those frequently obtained for soils.

The initial tangent moduli and failure strains are most dependent upon void ratios which, like frictional properties, are dependent upon individual grain roughness. In general, high surface roughness leads to high void ratios and high friction angles, but, for a given grain, intermediate void ratios lead to the highest friction angles and lowest failure strains.

The data obtained should prove useful as inputs for many finite element models of the stresses in grain silos.

SUMMARY AND CONCLUSIONS

The lateral stresses exerted by a granular material within a silo during unloading may be considerably higher than those exerted when the silo is filling. No coherent explanation has been advanced to explain this phenomenon, but it is hypothesized here that dilation of the material upon shearing is the cause of the overpressures.

Tests in a specially constructed, low confining stress triaxial test apparatus indicate that maize, wheat and barley dilate during standard triaxial shear tests, whereas oats, which has higher void ratios, does not. The tests also show that the Mohr failure envelopes are not linear, as normally assumed in silo design, but are concave downwards. The lower friction angles at higher confining stresses are attributed to the deformation of individual grains in the bulk material.

General expressions for the compression loading tangent modulus, and the Mohr failure envelope are derived, and the parameters for the four grains evaluated. These parameters are suitable for use in many finite element models of the stresses in silos. The shear strengths and initial tangent moduli are greatest, and the failure strains least, at the intermediate bulk densities of maize, wheat and barley. This may be explained by the greater degree of interlocking at the intermediate bulk densities. It is suggested that the data for oats did not follow the above trend, because of the higher void ratios involved.

Standard and constant volume tests indicate that stress-strain characteristics are most dependent upon grain packing and stress history, whereas frictional behavior is more dependent upon the geometry and surface roughness of the individual grains. In the constant volume tests, the tendency for dilation increases with decreasing void ratios, regardless of the characteristics of the individual grains.

Although particle characterization has not been adequate to predict the stress-strain behavior of soils in bulk, these tests indicate that the particle characterization of agricultural grain, which is a uniform, cohesionless material, may be more readily correlated to triaxial load response.

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APPENDIX A: DESIGN STRESSES IN SILOS - AN HISTORICAL REVIEW

The bulk storage of granular materials was made possible by the development of practical elevators and conveyors in the 1860s. The first designers believed that granular materials behaved like fluids, and their structures were designed to withstand hydrostatic pressure distributions. Experiments performed by Roberts (1882, 1884) with wheat and dried peas indicated that, unlike a fluid, a granular material exerted: i) a vertical shear stress on the walls of the structure, and ii) a lateral stress that did not increase linearly with depth.

Two types of stress distribution were found to exist. The classic theories of Rankine and Coulomb were satisfactory for predicting stresses in shallow bins, that is bins where the rupture plane cuts the grain surface, but they were inadequate for predicting stresses in deep bins.

Two deep bin stress prediction equations gained early acceptance for design purposes. Janssen (1895, 1896) took an analytical approach and summed all the forces acting on a horizontal element within the granular mass and set them equal to zero. The integration of the first order differential equation, which results from the statement of static equilibrium, yielded an expression for the vertical stress at any depth. The lateral stress was given as the product of the vertical stress and a lateral stress ratio. Koenen (1896) suggested using the active lateral stress ratio, $(1 - \sin\phi)/(1 + \sin\phi)$, where ϕ is the angle of internal friction, consequently Janssen's equation is also known as Koenen's solution in continental Europe.

Airy (1897) used a sliding wedge theory which is similar to that currently used in the design of retaining walls. A wedge of granular material was assumed to have commenced sliding along a plane, the strain mobilizing the intergranular friction along the shear plane and between the material and the bin wall. From the equations of static equilibrium, the critical failure plane could be defined, allowing the lateral stress at any depth to be determined without the use of a lateral stress ratio.

Shortly after Janssen's equation had been published, Prante (1896) reported increased lateral stresses when grain was unloaded from the bottom of the bin. Prante's results were subsequently believed to be too large because of the measuring apparatus employed (Ketchum, 1919). Other early investigators, Jamieson (1903), Bovey (1904), and Pleissner (1906) also reported increases in lateral stresses during unloading, while Toltz (1903), Lufft (1904), and Ketchum (1907) reported no increases. In his still highly respected textbook, Ketchum (1919) concluded from the results of previous workers that "The pressure of moving grain is very slightly greater than the pressure of grain at rest (maximum variation for ordinary conditions is probably 10 percent)," whilst, as the result of his own, subsequent experiments (undertaken as a check on the previous work), he concluded that "There was no appreciable increase in pressure for moving grain with a concentric discharge gate."

The use of Janssen's equations for bin design was frequently recommended (Hay, 1920, 1928; Long, 1931, 1932; Kelly, 1940; and Martin, 1940), and seems to have been satisfactory until the 1930s when refinements in design procedures and construction methods led to a reduction in safety factors. As a result of a number of structural failures, interest in stress prediction was renewed.

Dynamic overpressures, which are lateral normal stresses in excess of the at-rest bin wall normal stresses predicted by Janssen's equation (Manbeck et al., 1977), have been reported by numerous European workers. Much of this work was published in French or Russian, but has been summarized in English by a number of authors (Turitzin, 1963; Jenike and Johanson, 1968; Lumbroso, 1971; Garg, 1972, 1973; Britton, 1977; and Manbeck et al., 1977).

Takhtamishev, as cited by Turitzin (1963), measured lateral stresses during the unloading of grain elevators in 1938 and 1939, and reported lateral stresses between two and three times greater than those predicted by Janssen's equation. Takhtamishev's tests on concrete models produced lateral stresses 1.65 greater for sand and 1.35 greater for wheat in motion than for the same material at rest. Takhtamishev observed two general types of flow during his experiments. In the first type, which is now called nondynamic, core or funnel-flow, a central funnel of flowing material formed above the central discharge opening. The remaining material stayed stationary, except that at the surface which moved in to supply the funnel. In the second type, which is now called dynamic- or mass-flow, the entire mass moved simultaneously towards the discharge opening. It was found that mass-flow was responsible for much greater lateral stresses than funnel-flow. In many of the experiments funnel-flow occurred initially, but was succeeded by mass-flow when the bin was more than half empty. Takhtamishev was able to obtain funnel-flow by compacting the granular material evenly in the bin, and was able to obtain mass-flow by allowing the material to fall loosely into the bin.

M. and A. Reimbert (Reimbert, 1943) conducted many tests on models of deep bins between 1941 and 1943, which resulted in their publishing a new theory for predicting static granular stresses in deep bins. A hyperbola was adopted as a representative curve for the vertical shear stress developed by friction against the walls of the bin. The curve parameters were related to the bin geometry and the properties of the stored material. Coordinate geometrical constraints allowed for unique solutions for the parameters. Algebraic manipulation permitted the lateral stresses on the walls and vertical stresses on the bottom to be calculated. The active lateral stress ratio was used, although the Reimberts alluded to the fact that the ratio is not constant with depth. Sample calculations on typically sized bins by Turitzin (1963) indicated that the Reimbert theory predicted lateral stresses about 10% higher at the upper part of the bin, and about 6% lower at the lower part of the bin that those predicted by the Janssen equation.

No separate theory was advanced by the Reimberts for predicting stresses during emptying; instead, they recommended increasing the lateral stress values by a dynamic coefficient. As in Takhtamishev's experiments, the Reimberts found that the greatest lateral stresses occurred when loosely packed material emptied with mass-flow.

In order to induce funnel-flow rather than mass-flow, Reimbert and Reimbert (Reimbert, 1954) suggested the installation of a central, perforated tube extending from the top of the material to just above the outlet. When the discharge gate is opened the material inside the tube moves first and is replaced by material from the upper part of the silo, the bulk of the stored material remaining stationary. The Reimberts' explanation for this phenomenon is that, according to their theory, the lateral stress is proportional to the hydraulic radius of the container and, because the tube is much smaller than the bin, the material inside the tube is less compressed, and therefore flows out more readily than the material outside the tube. The effectiveness of their method was proved on models and full-size bins (Reimbert and Reimbert, 1956, 1976).

Jàky (1948) criticized the constant value of wall friction assumed in Janssen's equation, and offered a more general solution for the stresses developed within grain stored at rest in a silo. The vertical forces on an elemental disc were equated, and an expression for the coefficient of wall friction was obtained in terms of the material density, the hydraulic radius of the silo, and the slope of the stress-depth curve. Jàky showed from previously published data that the shear stress distribution at a silo wall consists of an upper zone in which the shear stress increases linearly, and a lower zone in which the shear stress remains constant. He stated that the lateral stress ratio varies, particularly in the upper zone, and is a function of the physical properties of the stored material and of the geometry of the silo. He further stated that wall friction increases rapidly with depth near the top of the silo until a maximum is reached, whereupon a lower value is approached asymptotically. Jàky pointed out that this curve is similar to the shearing stress-strain curve of a dense granular material, which he considered to be reasonable, since strain due to settling may be assumed to be proportional to depth. Jàky's solution assumes the uniform distribution of the vertical stresses within the material which produces a wall friction vs. depth curve similar to the shear stress-strain curve of a loose granular material. This assumption produced an overestimate of 7% for the vertical stresses, but yielded very close estimates of the lateral stresses in Jamieson's (1903) test.

Caquot and Kérisel (1949) and Caquot (1957) derived separate equations for predicting lateral stresses during the filling and emptying of silos. They considered the relationship between the horizontal and vertical forces on an infinitely small prism of granular, homogeneous, and noncohesive material. This relationship had the form of Rankine's active lateral stress ratio for filling, and Rankine's passive lateral stress ratio for emptying. For each equation, the horizontal and vertical stresses were considered as part of the system of forces in equilibrium acting on an infinitessimal slice of a cylinder. The equations, integrated over the height of the stored material, yielded stress equations in the form of exponential functions similar to that of Janssen's equation. Sample calculations on a typical concrete bin by Turitzin (1963) indicated that Caquot and Kérisel's equations were in close agreement with Janssen's equation for the lower third of the bin, but predicted high lateral overpressures for the upper third.

Kim, as cited by Turitzin (1963), performed numerous experiments from 1948 to 1953 for the purpose of developing methods to prevent cracks in grain elevators, which he believed were caused by dynamic stresses during emptying. Observations were made simultaneously inside from a hoist and outside through the windows in the bin wall. Kim, like Takhtamishev, reported on the funnel- and mass-flow regimes, but he also noted an occasional mixed-flow type. Large lateral stresses were observed only during mass-flow. Kim concluded that there were two reliable methods of obtaining funnel-flow. The first was the installation of tubes, as suggested by the Reimberts. The second method was the installation of internal rings at several heights on the silo wall to prevent the stored material from moving towards the discharge gate en masse. He also concluded that a densely packed material does not in itself ensure funnel-flow.

Ohde in 1950 and Nanninga in 1956, both cited by Jenike and Johanson (1968), recalled an attempt made by Kötter in 1899, as cited by Gaylord and Gaylord (1977), to establish more rigorously the stress distribution within the stored material. Kötter suggested that active lateral stresses developed during filling of the bin, whilst passive lateral stresses developed during flow. During discharge an active-passive switch occurred at the plane of transition between the active stress field, in the upper portion of the bin, and the passive stress field in the lower portion. Nanninga developed formulae for the active and passive lateral stress ratios by assuming that the shear increased linearly from zero at the axis of the bin to a maximum at the wall. He noted that the equilibrium of vertical forces on the mass at the switch required an overpressure on the wall, and that the peak lateral stress decreased rapidly below the switch, approaching Janssen's value asymptotically.

Lateral stresses continued to be investigated (Amundson, 1945; Hool and Johnson, 1947; Caughey et al., 1951) in model and full-size silos but, since no stress increases upon discharge were found, Janssen's equation continued to be recommended. Amundson recommended a lateral stress ratio of 0.5 for all materials; Hool and Johnson recommended designing all grain silos using the internal angle of friction and bulk density of wheat; Caughey et al. reported the frictional characteristics of various grains for use in conservative designs.

Turitzin (1952) and Rogers (1952) emphasized the difference between the design procedures for deep and shallow coal bunkers. Rogers recommended that the classic Coulomb theory be used for large, shallow bunkers, and that the Janssen equation be used for large, deep bunkers.

Jakobson, as cited by Britton (1977), developed a differential equation to predict lateral stresses, assuming that all granular material settled vertically in the bin. It agreed closely with Janssen's equation when the appropriate boundary conditions were applied.

In 1959 Kovtun and Platonov, as cited by Turitzin (1963), developed automatic instrumentation for measuring and recording stresses in deep bins. It was observed that, during filling, the normal lateral stresses increased gradually with depth, and were only slightly greater than those predicted by Janssen's equation. Upon opening of the discharge gates, however, a considerable increase in normal lateral stress, as much as 2.32 times, was observed. During emptying, both lateral and vertical stresses were of a pulsatory nature, the effect being greatest near the walls. As a result of their findings, Kovtun and Platonov proposed a scheme for determining dynamic stresses in which the bin was divided into three zones. Stresses were calculated in the upper zone by using the Rankine solution increased by a safety factor of 1.65. From equilibrium considerations they determined that the lateral stress in the middle zone was equal to one half the product of the bin diameter, the unit weight of the stored material and the coefficient of internal friction. It was found that this latter varied with the material density which, in turn, was a function of the stored material height. The stresses in the lower zone, although not well-defined, were assumed to decrease linearly as a result of stress transfer by the material immobilized against the sides and bottom of the bin.

The observation of dynamic overpressures cast serious doubts as to the reliability of Janssen's theory. Geniev, as cited by Turitzin (1963), took an analytical approach to explain the phenomenon of dynamic stress increases. He claimed that Janssen's theory was incorrect, since it did not satisfy boundary conditions and assumed uniform vertical stress distribution. Geniev assumed the vertical stress distributions to be parabolic, and determined a dynamic stress curve for the emptying of a non-compressive, ideal granular material with constant internal coefficent of friction. Geniev concluded that these assumptions were invalid since his theory predicted lateral stresses that were, at the most, only 1.25 times those predicted by Janssen's theory. Geniev then assumed that the granular material had a density which increased with depth and an internal coefficient of friction which varied with density. The latter function was an important consideration, since his tests had shown that density changes in the order of 5% resulted in the doubling of the angles of internal and wall friction. Geniev derived an equation for the moving particles which was similar to Bernoulli's equation for the flow of liquids. Lateral stresses 2.05 times larger than those predicted by Janssen's equation were obtained for the wall at its junction with the hopper (equivalent to the junction of the middle and lower zones of the flat bottomed bins considered by Kovtun and Platonov).

Pameland (1959) explained the lateral stress increases during discharge in terms of a rapid decompression of the material in the lower portion of the bin producing a series of self-supporting domes which collapsed immediately upon formation. He assumed a parabolic lateral stress distribution, and derived a formula which he increased by a factor of 1.5 to account for dynamic overpressures. He compared his formula in dimensionless terms with those of Janssen and Reimbert, and pointed out that his formula was both the simplest and the most conservative. He conceded that his formula should serve only as a guide, since material behavior is altered by even small variations in its properties.

Jenike (1961, 1964) developed differential equations for flow-induced stresses in a radial stress field, i.e. a field in which stresses increase linearly from the vertex of the hopper. He assumed that wall friction was fully developed and that the granular material flowed in a plastic-passive state. The conjugate stress ratio was given by $(1 - \sin\delta)/(1 + \sin\delta)$, where δ is the effective or dynamic angle of friction, found by experiment, and assumed to remain constant for a given material. Later Jenike and Johanson (1968) published differential equations for initial (static) loadings, again assuming a radial stress field and that wall friction was fully developed. The material was assumed to be in an elastic-active state and, after preliminary experimentation, the conjugate stress ratio was tentatively assumed to be constant. Jenike and Johanson pointed out that it was not sufficient to design only for the larger of the stress distributions, but that the stresses produced by the transition from the active to the passive state should be accommodated. When the discharge gate is opened, the switch from active to passive state travels up the bin, causing overpressures at its contact with the wall due to the unsupported weight of the material in transition. It was reported that, in the case of a conical hopper below a cylindrical bin, the switch may be permanently arrested at the change in section, or the switch may proceed up the cylindrical section, with the overpressure initially increasing in magnitude before decaying to zero at the top of the bin. Depending upon the size of the bin, the compressibility of the solid, and the rate of discharge, the switch may last a fraction of a second, producing a shock load, or it may last much longer.

Jenike and Johanson assumed the material to be fully plastic at the transition from active to passive state, and developed an upper bound to the overpressures using Sokolovski's solution. The values obtained were extremely high (an overpressure 45 times that of the static pressures for a wall friction angle of 20°, for example). They conceded that such high pressures could not be sustained by grain, but suggested that they could be sustained locally by rock or gravel. It was further suggested that very high local pressures were responsible for the diversity of results obtained by experimenters using large pressure transducers.

Jenike and Johanson explained the criteria for mass-flow in terms of the development of a radial stress field, and then used them to explain the frequently observed change from mass-flow to funnel-flow during emptying.

Collins (1963) discussed the influence of material properties on the stresses predicted by Janssen's equation, and examined the behavior of wheat in model, paper silos. He observed that the horizontal stresses and vertical shear stresses on the wall were increased during discharge of the wheat, as was evidenced by the bulging and buckling of the paper.

Pieper and Wenzel, as cited by Fankhauser (1977), conducted experiments to validate Janssen's equation. They concluded that the theory and equation were valid provided that the lateral stress ratio was defined as $(1 - \sin\phi)$ as suggested by Jàky (1948).

Philips (1965) concluded from Pieper and Wenzel's experiments that vertical stresses were greatest during filling and storage, but that lateral stresses were greatest during emptying. He recommended the use of the German standards DIN 1055, Part 6 for design. These design regulations were based on Janssen's equation, but lateral stress ratios were fixed as 0.5 for filling and 1.0 for emptying, and the coefficients of wall friction were defined as $\tan(0.75\phi)$ for filling, and $\tan(0.60\phi)$ for emptying.

Yamahara and Takahashi, as cited by Manbeck et al. (1977), observed dynamic overpressures 2.5 times as great as the static stresses in a concrete bin. They recommended use of the Soviet Code, CH-302-65, which is based on Janssen's equation, with the lateral stress ratio and the coefficient of wall friction fixed at 0.44 and 0.40, respectively. Separate correction factors were applied to the equation for the upper third and lower two thirds of the bin to account for overpressures (due to temperature changes, pneumatic conveying, discharge etc), construction, and shape.

Isaacson and Boyd (1965) presented a general mathematical analysis of lateral stresses in bins. They suggested that a deep bin be defined as a bin in which at least 95% of the wall friction is developed at some level above the bottom; for circular grain bins this leads to a height to diameter ratio greater than or equal to about 5. They showed that the fundamental behavior of the one-dimensional, grain bin stress mechanism may be characterized by an ordinary linear differential equation, and proceeded to give the general solution. When the boundary conditions were applied, the solution reduced to Janssen's equation, if the parameters were held constant, and to Reimberts' semi-empirical solution, if the lateral stress ratio was assumed to decrease hyperbolically with the other parameters held constant. It was pointed out that Janssen's and Reimberts' solutions, which previously had been thought to have been independent, were special cases of the general solution which could itself be used to advantage for static and dynamic equilibrium if the functions could be determined experimentally.

Later, Isaacson (1971) developed a computer-based mathematical model which allowed for variable density, friction and lateral stress ratio functions in the general solution. He developed the one-dimensional solution into two dimensions and presented examples of numerical and graphical solutions for lateral stress distribution. Isaacson alluded to the interdependency of the functions and its implications, particularly under dynamic conditions, and outlined an iterative relaxation solution. He also discussed topological models based on grain-pile transformations, where the stored material was assumed to have a tendency to collapse into a freestanding pile. A numerical technique, described as a hybrid between traditional digital simulation and finite-difference techniques, was used to develop dynamic lateral stress ratio functions.

Walker (1966) developed an approximate theory for stress prediction in coal hoppers. He took a similar approach to that of Jenike, and assumed that during filling and storage an active Rankine stress field existed, but he calculated the lateral stress ratio using the effective angle of friction. Stress fields during mass-flow were derived by considering the forces acting upon horizontal elemental slices which yielded both within themselves and along the hopper walls. Walker showed that the dynamic pressures may be calculated from Janssen's equation, where the product of the lateral stress ratio and coefficient of wall friction is given by a trigonometric relationship involving the effective angle of internal friction, the angle of wall friction, and, in the case of hoppers, the angle of the wall to the vertical.

Safarian (1969) presented a design procedure using Janssen's or Reimberts' equations modified by correction factors based on the Soviet Code and the results of Pieper's experiments. The bin was divided into three horizontal zones. The stresses in the upper zone were calculated using

Reimberts' solution, or by 1.35 times Janssen's solution. The stresses in the lower zone, which comprised the lower two thirds of the bin, were calculated using the Janssen or Reimbert equations in conjunction with his empirical overpressure factors. A linear stress distribution between the boundaries of the upper and lower zone was assumed for the intermediate zone.

In 1968, the American Society of Mechanical Engineers held a materials handling conference, at which a number of authors presented their contributions to the state of the art at that time. Jenike and Johanson (1969) outlined their theories of initial, flow and switch loading, based on the evaluation of stresses in active and passive stress fields, and Johanson (1969) related these loadings to the formation and collapse of arches in mass- and funnel-flow bins.

Theimer (1969) reported upon failures in concrete grain silos, and pointed out that many were due to the inability of designers to predict stresses induced by the storage and discharge of grain. He discussed the values assumed by various authors and standard codes for the friction angles and for the lateral stress ratio during static and flow conditions. Theimer recommended using Janssen's equation and a safety factor varying with height between 1.0 and 1.5 for internal walls, and 1.0 to 2.0 for external walls.

Deutsch and Schmidt (1969) reported upon their studies of the stress characteristics of sand in a model silo. Overpressures up to four times the static stresses during mass-flow were observed in certain regions of the silo. There was no evidence of overpressures being a function of flow rate. Deutsch and Schmidt proposed a conservative design curve which consisted of three portions of linear stress distribution.

Pieper (1969) reported upon investigations undertaken to verify the German Standards concerning silos, DIN 1055 Part 6. Two model silos were used to investigate the nature of the lateral stress ratio, and the material-wall friction during filling and discharge of the silo. The rates of filling and discharge were found to have no effect on the stresses produced, however eccentric discharge was found to cause larger stresses and these were studied in some detail. Pieper concluded that, although the German Standards were incorrect in some details, silos designed according to the specifications should not be damaged.

Moody (1969) presented the state of the art of the design of storage tanks for plastic powders. Airy's and Janssen's formulae with conservative safety factors were recommended for estimating lateral stresses. No specific recommendations were made to allow for overpressures developed during discharge.

Lvin (1971) developed dimensionless differential equations for predicting stresses in deep bins. The use of elemental rings, rather than the elemental discs of Janssen, allowed a more general solution which did not assume a uniform vertical stress at each horizontal section. In contrast to Janssen's solution, Lvin's solution revealed the existence of two stress zones; an upper zone in which the lateral stresses increased parabolically up to a maximum coincident with the asymptotic, limiting value in Janssen's equation, and a lower zone in which the stresses were constant and equal to the maximum value attained in the upper zone. Lvin modified his static solution to account for dynamic overpressures, by assuming an active lateral stress field in the upper stress zone, and a passive stress field in the lower zone. No discontinuity was incurred at the boundary between the zones, since the asymptotic value of lateral stress in Janssen's equation is independent of the lateral stress ratio. Lvin's dynamic solution allows for a lateral stress ratio that varies linearly with depth in a transition zone between the stable material above, and the rupturing material below. This had the effect of increasing the maximum lateral stress, the increase being greater for larger, passive lateral stress ratios, i.e. for materials with higher angles of internal friction.

Lumbroso (1971) outlined the existing methods for numerically determining the loads applied by materials stored in silos. He suggested that stresses in the upper regions of deep bins are a linear function of depth, and should be estimated from the retaining wall theory used for shallow bins, rather than from the asymptotic theories, such as those of Janssen, Caquot and Kérisel, Reimbert etc. He further suggested that a bin should only be considered deep, and therefore suitable for design by the asymptotic theories, if the height to hydraulic radius is greater than 5 (i.e. for a circular silo, a height to diameter ratio greater than 1.25), and if the diameter does not exceed 20 meters. Lumbroso alluded to the dangers of ignoring the dependence of friction angles and bulk density upon confining stresses, and recommended using a "conventional angle of internal friction" and a mean bulk density, together with particle-behavior coefficients for filling and emptying, to calculate lateral stresses according to the theory of Caquot and Kérisel. This recommendation is applicable only to deep, rough-walled (concrete) silos, since a direct relationship between the tangents of the internal and wall friction angles was assumed.

Reisner and Rothe (1971) collated information on the handling of bulk materials from various disciplines, and presented the state of the art in the first comprehensive textbook since that of Ketchum (1919). The current design procedures and theories of lateral stress were presented with a preference for the theories of Jenike and Johanson. An extensive bibliography on bulk material handling, storage and testing was included.

Perry and Jangda (1971) measured stresses within the stored sand and at the walls of model silos, using an implanted "radio pill" pressure transducer. They alluded to the beneficial first in - first out characteristic of mass-flow bins, and concluded that the inevitable overpressures that these bins incur could be adequately predicted by the complete theory of Jenike and Johanson, or by the simplified theory of Walker.

In 1972, the American Society of Mechanical Engineers held a second materials handling conference, where Jenike et al. (1973a, b, c) presented the latest developments in their theories on bin loads. The authors developed equations to determine the lateral stress distribution in the cylindrical portion of mass-flow and funnel-flow bins. They assumed that the recoverable strain energy of the stored material below the switch from active to passive stress field would be minimized during steady state flow. This stress distribution was shown to be an upper bound, whereas Janssen's equation gave the lower bound. It was found that the stresses in mass-flow bins normally oscillate between these two bounds due to imperfections in the bin geometry, however a slightly divergent bin ensured that stresses remain at the lower bound. A constant value of 0.4 was recommended for the lateral stress ratio for all materials, and dimensionless charts were presented for determining the stress distributions in cylindrical bins and hoppers using this value. Wright (1973 a, b) conducted a number of experiments with iron ore to evaluate the Jenike-Johanson design procedure. He concluded that the measurement of the stored material properties led to the adequate design of hoppers operating under dynamic flow conditions, and that the criteria set down by Jenike (1964) to delineate between mass-flow and funnel-flow were correct. He noted, however, that the design procedure did not guard against stresses due to impact loading, or ensure a free flowing hopper if there was a surcharge load from material above the hopper, or if the material gained strength during prolonged storage. Clague and Wright (1973) evaluated Walker's theory in a similar manner and came to similar conclusions.

Garg (1973) evaluated the German and Soviet design codes in the light of internationally reported experimental work, and concluded that the Soviet code, which had been adopted by Japan, was overly conservative, whereas the German code was adequate and, with slight modifications, had formed the basis of the Indian code, IS:4995-1968. Later experimental work (Garg and Gopalakrishnan, 1974) with wheat in a model silo, indicated that this code too was inadequate, and a modification was suggested to account for the large overpressures which appear to be those due to the switch in the Jenike-Johanson theory.

Cornish (1973) analyzed the stresses developed at the junction of a bin and a converging hopper. Using the theory of stress discontinuity set out by Schofield and Wroth (1968), Cornish showed that higher stresses than those predicted by the theories of Jenike and Walker could develop during mass-flow. The estimated stresses were higher for materials with high effective angles of internal friction, and for bins with smooth, steep hopper walls, the bin characteristics required to promote mass-flow.

McLean and Arnold (1976) clarified the Jenike and Johanson design procedure, which requires considerable numerical computation, by assuming a single bound approximation. Using this assumption, a single calculation of strain energy stress yielded peak stresses which were in close agreement with Jenike and Johanson's more elaborate estimates.

Richards (1977), Van Zanten et al. (1977), Van Zanten and Mooij (1977) and Everts et al. (1977) made stress determinations on model silos containing P.V.C. powders and sand to evaluate current design procedures. Richards confirmed the accuracy of Jenike's criteria for flow regime (i.e. mass-flow vs. funnel-flow) and free flowing outlets, which are based on material properties and bin geometry. Van Zanten and Mooij found that overpressures, which could be in excess of the equivalent hydrostatic stresses, were developed above induced irregularities in the cylindrical portion of the silos. These overpressures were attributed to local passive bracing. They found that Janssen's equation predicted adequately the stresses in the upper portion of the mass-flow silos, but that Jenike and Johanson's strain energy model was more applicable near the hopper, where large lateral stresses were observed during discharge. The strain energy model was also found to be suitable for funnel-flow bins.

In 1977, the American Society of Agricultural Engineers held a conference at which a number of authors presented theoretical and experimental work on the prediction of lateral stress in agricultural storage bins. Gaylord and Gaylord (1977) discussed the nature of active and passive stress fields within storage bins, and compared various stress prediction equations with published experimental data.

Moysey (1977) pointed out that the use of the passive lateral stress ratio in place of the active lateral stress ratio was inadequate for estimating overpressures since, contrary to experimental observations, the largest stress increases are predicted in the upper portions of the bin. Moysey favored Lvin's (1971) approach for estimating dynamic stresses. Manbeck et al. measured stresses in model bins designed according to the principles of dimensional analysis. They concluded that no significant overpressures developed during the emptying of bins less than 1.25 diameters in height, but that overpressures up to 3.89 times as large as static pressures were possible during the emptying of bins greater than five diameters in height. Fankhauser (1977) discussed a number of stress theories and design codes, and recommended using the German Code in designing corrugated grain tanks.

In their recommendations for design, the American Concrete Institute (1977) advocated the use of either Janssen's or Reimberts' theories in conjunction with overpressure and impact factors, for calculating design stresses in concrete silos. They cautioned that these factors were inadequate for the design of mass-flow silos. Kothari (1979) commented on the application and limitations of the recommendations for calculating stresses in shallow bins and hoppers.

Vivancos (1978) generalized Reimberts' theory by assuming that an active stress field existed during filling and storage of a silo, and that a passive stress field existed during emptying. In common with the use of active and passive lateral stress ratios in Janssen's equation, this theory leads to only small lateral stress increases in the lower portions of the silo. Calculated and observed lateral stresses during filling and emptying were in good agreement for the one silo examined. This silo appears to be of the funnel-flow type, since no large overpressures were evident.

Smith and Lohnes (1980 a, b) hypothesized that dilation was the cause of overpressures. Triaxial test data on maize were presented to support the hypothesis, but no design criteria were given. Marchant (1980) developed an incremental stress-strain model for granular materials. This model, which included a dilation component, was developed for the finite element analysis of the filling and storage stresses in shallow grain bins. Marchant's model predicted well the behavior of wheat over a number of stress paths in a triaxial test apparatus.

In conclusion, the practical design of most silos is still based on Janssen's (1895, 1896) formula using the active lateral stress ratio, or upon Reimbert's (1943) formula. For applications where the flow regime is important, Jenike and Johansons's (1968) criteria and design procedures are gaining acceptance (Wood, 1981).

APPENDIX B: DETAILS OF THE TRIAXIAL APPARATUS

The triaxial apparatus, illustrated in Fig. B1, consists of six basic components:

i) a tension-compression machine with continuously variable drive for the application of axial stresses,

ii) a specially constructed triaxial cell suitable for performing standard (constant lateral stress), constant volume, and zero lateral strain tests with lateral stresses up to 70 kPa,

iii) a volume monitor for maintaining constant the pore air volume during constant volume tests, and the cell air volume during zero lateral strain tests,

iv) a volumeter for measuring the change in pore air volume during standard tests,

v) a mercury manometer for measuring lateral stresses, and

vi) a control panel containing switches and valves to control the components described above.

Components i, v and vi require no special comment, but details of the other components are given below.

The triaxial cell is illustrated in Fig. B2. The top and bottom plates and the piston guide were machined from aluminum, the lower and upper end platens, the piston, and the surrounding cylinder were machined from Plexiglas, and the remaining components were machined from steel or brass. To the piston is attached a 120 mm diameter proving ring, to which in turn may be attached a spacer for compression testing, or a 900 kN, 12 volt electromagnet for compression and extension testing. A steel

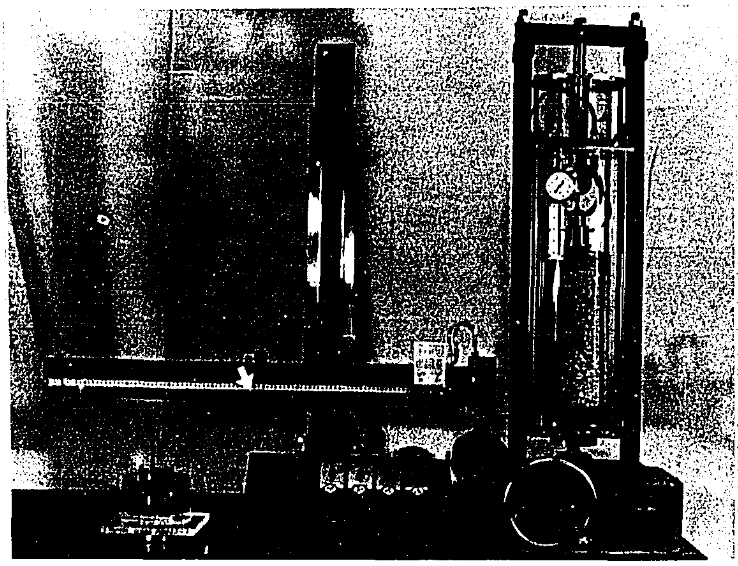

Fig. B1. Triaxial test apparatus

ball is placed in a recess in the top platen when the spacer is used; a ball-and-socket joint is attached to the top platen when the electromagnet is used. A Neoprene O-seal, lubricated with vacuum grease, provides an air-tight seal between the piston and the top plate.

The volume monitor consists of a 3 mm inside diameter glass tube, containing a bead of mercury approximately 15 mm long. In operation, an electrically driven agitator vibrates the tube, improving the response of the mercury to pore air or cell air pressure changes due to changes in the specimen volume. To facilitate the retention of the mercury bead close to its initial position, the glass tube is positioned over a scale. In all tests it was possible to keep the mercury bead within 100 mm of its initial position, which corresponds to maintaining constant specimen

Fig. B2. Triaxial cell containing a maize specimen

volume to within 0.02%.

The volumeter consists of an inverted, thin shell (0.08 mm) copper cup floating in water within a support (see Fig. B3). As the specimen volume changes during a standard test, pore air is drawn in or expelled from the specimen, causing the copper cup to raise or fall relative to a scale. The volumeter was calibrated against precision 200 ml gas syringes accurate to within 1 ml, which is approximately 0.02% of the specimen volume.

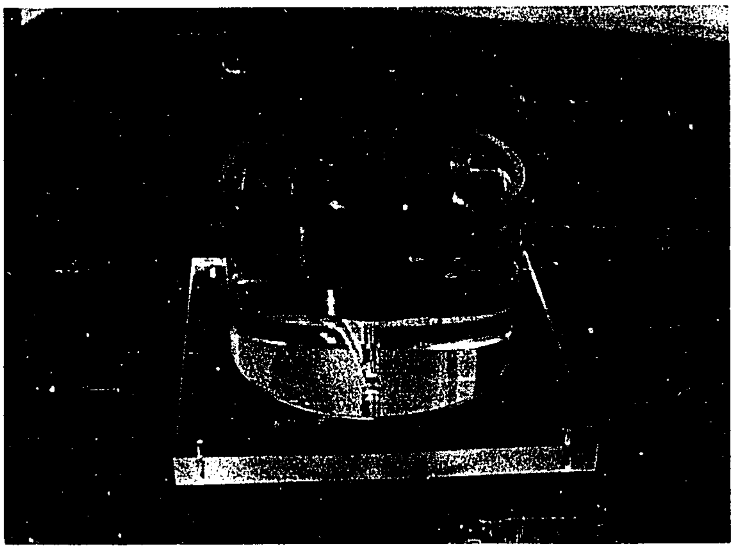

Fig. B3. Volumeter

APPENDIX C: MICROGRAPHS OF GRAIN SURFACES

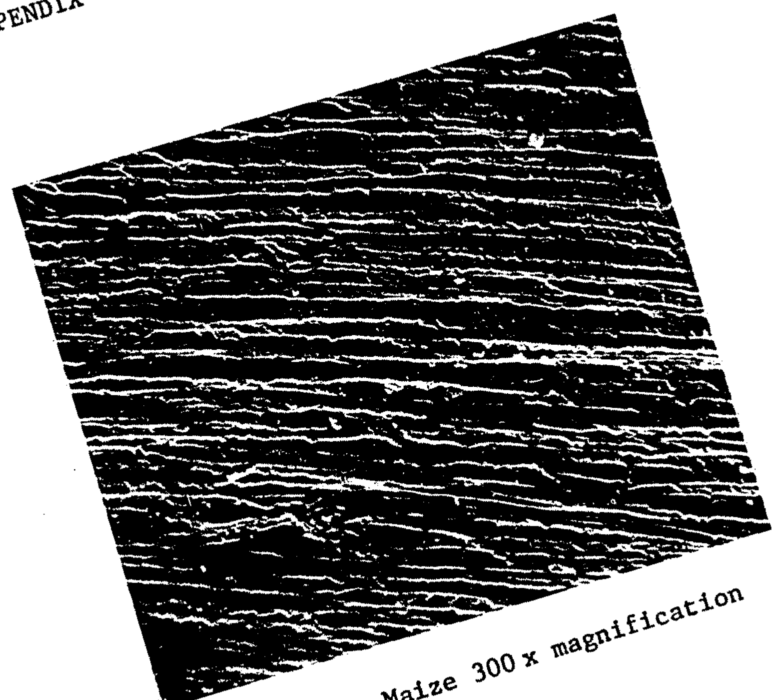

Fig. C1. Maize 300 x magnification

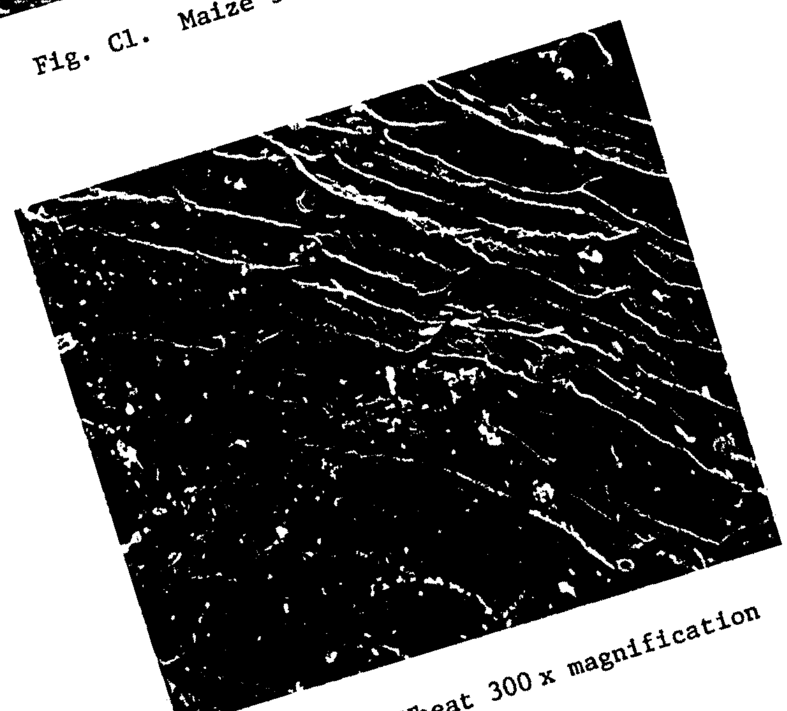

Fig. C2. Wheat 300 x magnification

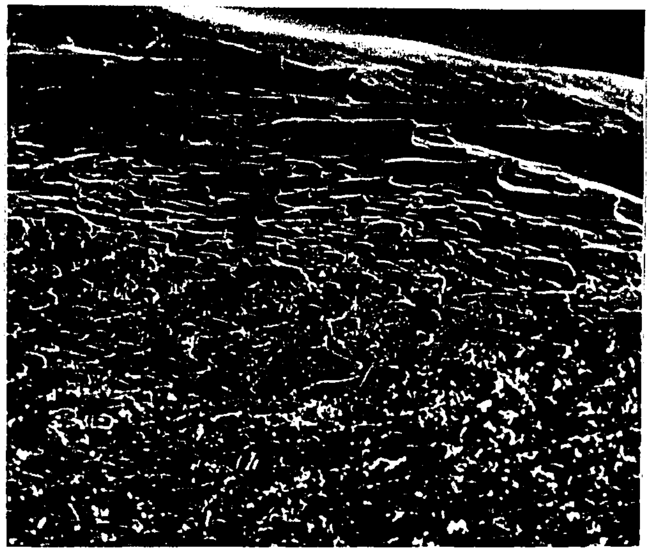

Fig. C3. Barley 300 x magnification

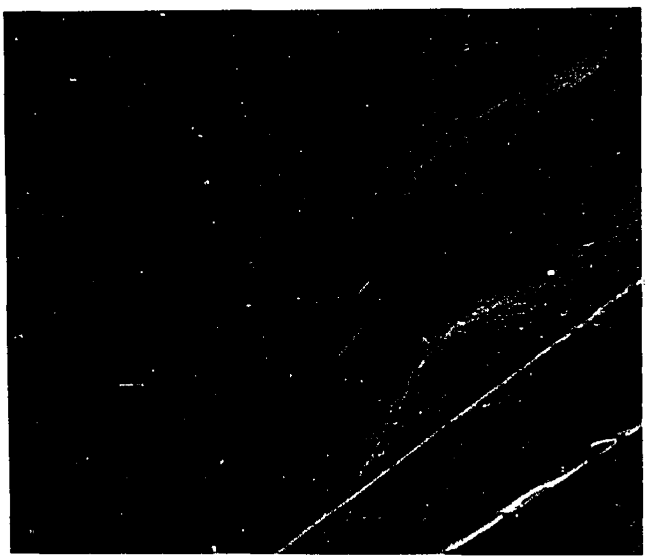

Fig. C4. Oats 300 x magnification

APPENDIX D: NUMERICAL APPROXIMATION OF MOHR FAILURE ENVELOPES

For a cohesionless material with a linear Mohr envelope, it can be shown (Lambe and Whitman, 1979) that:

$$\sin \phi = q/p$$

where ϕ = the internal angle of friction (i.e. $\tan \phi$ = slope of Mohr failure envelope)

(p, q) are the coordinates of the top of the failure circle

For a material with a nonlinear Mohr envelope (see Fig. 2, part IV) then:

$$\sin \phi_1 = \left[\frac{q_i - q_{i-1}}{p_i - p_{i-1}}\right]$$

where ϕ_i = the angle between the σ axis and the line tangent to the i^{th} and $(i-1)^{th}$ circles

(p_i, q_i) are the coordinates of the top of the i^{th} failure circle

The point of tangency of this line is given by the coordinates $(\phi_{i.1}, \tau_{i.1})$, where $\phi_{i.1} = p_i - q_i \sin \phi_i$ and $\tau_{i.1} = q_1 \cos \phi_i$.

$(\sigma_{i.2}, \tau_{i.2})$, the coordinates of the point of tangency on the i^{th} circle of a line tangent to the i^{th} and $(i+1)^{th}$ circles are given by:

$$\sigma_{i.2} = p_i - q_i \sin \phi_{i+1}$$

$$\tau_{i.2} = q_i \cos \phi_{i+1}$$

The desired coordinates, (σ_{fi}, τ_{fi}), for the points of tangency of the non-linear Mohr failure envelope are approximated by:

$$\sigma_{fi} = \tfrac{1}{2}(\sigma_{i.1} + \sigma_{i.2})$$

$$\tau_{fi} = \tfrac{1}{2}(\tau_{i.1} + \tau_{i.2})$$